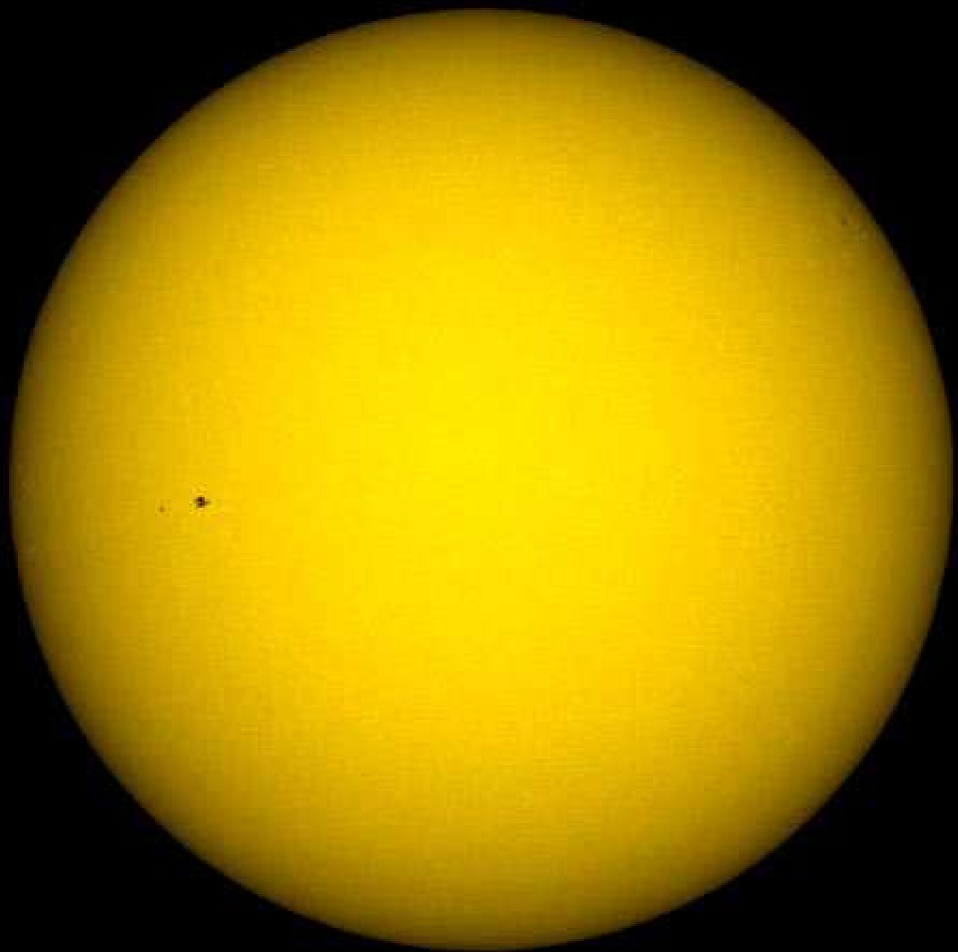
Neutron Stars

Jörn Wilms
Dr. Remeis-Observatory & Erlangen Centre for Astroparticle Physics
University of Erlangen-Nuremberg
http://pulsar.sternwarte.uni-erlangen.de/wilms

Structure of this lecture:

- Neutron Stars: Introduction
  - Stellar Evolution
  - End Stages of Stellar Evolution
  - Structure of Neutron Stars
- Neutron Stars: Radio Pulsars
  - Discovery
  - Radiation Mechanism
  - Radio Pulsars as a Class
  - Testing Relativity: Binary Pulsars
- Neutron Stars: X-Ray Binaries
  - Neutron Stars in Binary Systems
  - Continuum formation in the accretion column
  - High Mass X-ray Binaries
  - (Low Mass X-ray Binaries)



The Sun: A typical star

Stars: Gas balls in hydrostatic equilibrium

The structure of stars is determined by a set of four coupled differential equations which express the basic conservation and transport quantities always encountered in physics:

1. Mass conservation
2. Momentum conservation (=hydrostatic equilibrium)
3. Energy conservation
4. Energy transport

and quantities expressing the physical properties of material, mainly:

1. Equation of state (=dependence of density of material from physical conditions)
2. Energy generation

Stars: Gas balls in hydrostatic equilibrium

The structure of stars is determined by a set of four coupled differential equations which express the basic conservation and transport quantities always encountered in physics:

Mass structure
(mass conservation)

$$\frac{\mathrm{d}M}{\mathrm{d}r} = 4\pi r^2 \rho(r)$$

Pressure structure
(hydrostatic equilibrium)

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\rho(r)\frac{GM(r)}{r^2}$$

Temperature structure
(e.g. radiative transfer)

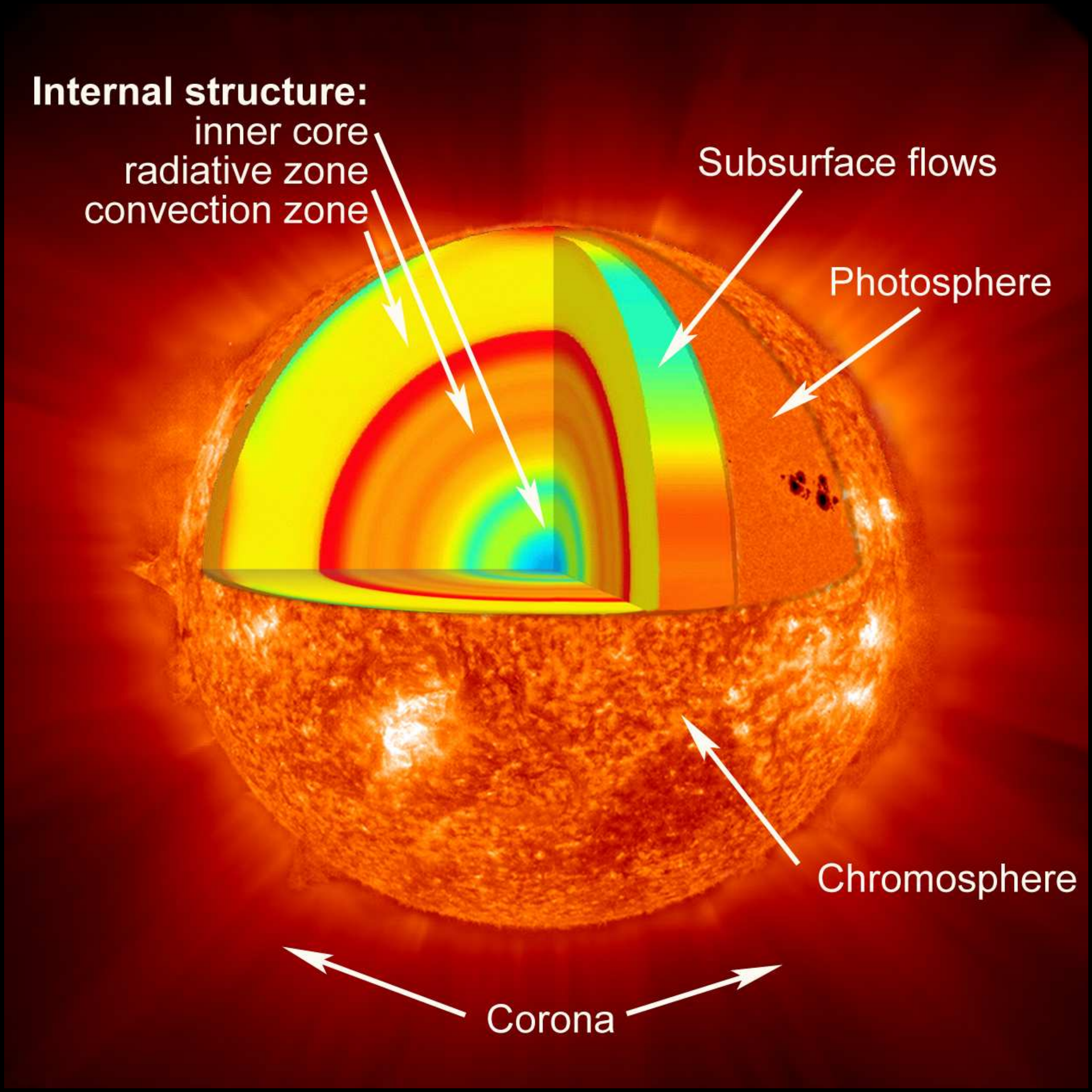
$$\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{3}{4ac}\frac{\kappa\rho(r)}{T^3}\frac{L(r)}{4\pi r^2}$$

Energy conservation

$$\frac{\mathrm{d}L}{\mathrm{d}r} = 4\pi r^2 \rho(r)\epsilon(r)$$

and quantities expressing the physical properties of material, mainly:

- "equation of state", $P = P(T, \rho)$,
- energy generation, $\epsilon = \epsilon(T, \rho, Z)$
- Opacities $\kappa(T, \rho, Z)$ = interaction of radiation with gas,



NASA

Stellar model: solution of structure equations

During normal stellar life: Gravitation balanced by thermal pressure:

$$P_{\rm g} = \frac{F_{\rm g}}{4\pi r^2} \tag{1}$$

Estimate P: Assume $\rho = \text{const.}$. Central Pressure = $\sum$ of gravitational force of thin shells over whole star:

$$P_{\rm g} = -\int_0^R G\frac{\left(4\pi r^2\rho \mathrm{d}r\right)\cdot\left(\frac{4}{3}\pi r^3\rho\right)}{r^2}\cdot\frac{1}{4\pi r^2} = -\frac{2}{3}G\rho^2 R^2 \tag{2}$$

$$\text{with } \rho = M/\left(\frac{4}{3}\pi R^3\right): \quad P_{\rm g} = -\frac{3}{8\pi}G\frac{M^2}{R^4} \tag{3}$$

Normal star: Gravitational pressure blanced by internal (thermal) gas pressure:

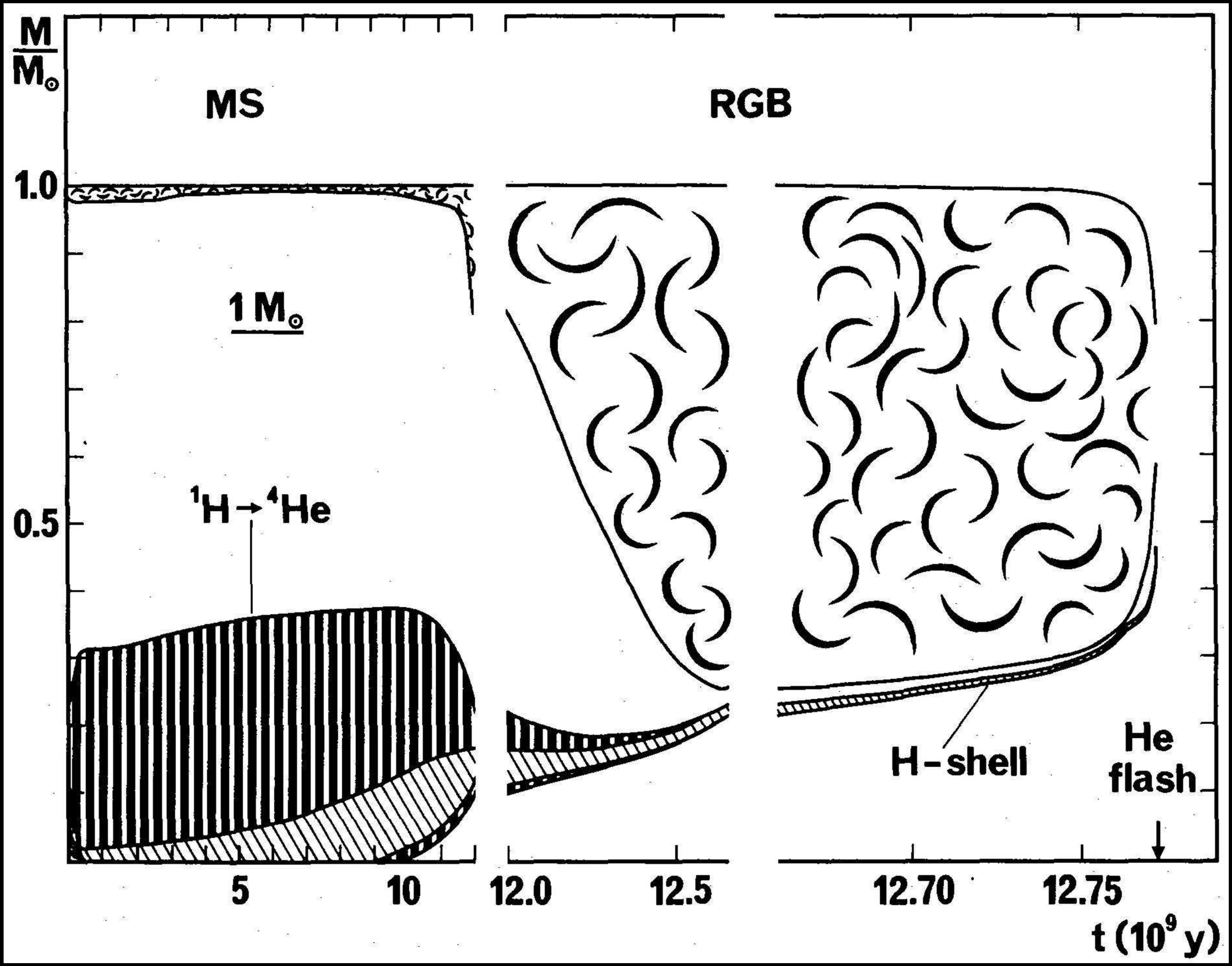
$$P_{\rm i} = nkT \sim \frac{M}{\frac{4}{3}\pi R^3\cdot m_{\rm H}}kT \tag{4}$$

Hydrostatic equilibrium $\Longrightarrow P_{\rm i} = -P_{\rm g}$

$$\frac{M}{\frac{4}{3}\pi R^3\cdot m_{\rm H}}kT = \frac{3}{8\pi}G\frac{M^2}{R^4} \quad\Longrightarrow\quad M = \frac{2kT}{Gm_{\rm H}}R \quad\Longrightarrow\quad R = \frac{Gm_{\rm H}M}{2kT} \tag{5}$$

$\Longrightarrow$ To keep star stable, vary radius (=density) and temperature.

This way, nuclear reactions are regulated and stars remain in equilibrium.



Evolution of the structure of a 1 $M_\odot$ star to the Helium flash (Maeder & Meynet, 1989).

If nuclear fuel is exhausted, no energy input into gas

$\Longrightarrow$ Star collapses, generates energy by gravitational contraction.

$\Longrightarrow$ Density increases until ideal gas law is not appropriate since QM effects become important (Electrons are Fermions!)

This is the case when the Fermi energy $\sim$ avg. thermal energy of electrons.

Fermi energy (for H-gas):

$$\epsilon_{\text{Fermi}} = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3} = \frac{\hbar^2}{2m}\left(3\pi^2\frac{\rho}{m_{\text{H}}}\right)^{2/3} \tag{6}$$

If $\frac{3}{2}kT < \epsilon_{\text{Fermi}}$: electrons are unable to transit into unoccupied state:

$$\frac{T}{\rho^{2/3}} < \frac{\hbar^2}{3m_{\text{e}}k}\left(\frac{3\pi^2}{m_{\text{H}}}\right)^{2/3} \sim 10^5\,\text{K}\,\text{cm}^2\,\text{g}^{-2/3} \tag{7}$$

or expressed as electron particle density

$$n > n_{\text{crit}} = 5 \times 10^{16}\,\text{cm}^{-3} \cdot T^{3/2} \tag{8}$$

Estimate: For $T = 10^7$ K, $n_{\text{crit}} \sim 10^{27}\,\text{cm}^{-3}$. For an object with $M = 1\,M_\odot$ this density corresponds to $R = 10^4$ km, i.e., Earth-sized.

Pressure of a degenerate electron gas:

Remember your 1st semester:

$$P = \frac{1}{3} n p v \tag{9}$$

Typical electron separation in the degenerate electron gas:

$$\Delta x \cdot \Delta y \cdot \Delta z = (\Delta x)^3 = \frac{1}{n_\mathrm{e}} \quad \Longrightarrow \quad \Delta x \sim n_\mathrm{e}^{-1/3} \tag{10}$$

Since electrons are densely packed: Heisenberg!

$$\Delta p \cdot \Delta x \sim \hbar \quad \Longrightarrow \quad p = \Delta p \sim \frac{\hbar}{\Delta x} = \hbar n_\mathrm{e}^{1/3} \tag{11}$$

and therefore with $v = p/m_\mathrm{e}$:

$$P_\text{degen, nonrel} = \frac{1}{3} n_\mathrm{e} \cdot \hbar^2 n_\mathrm{e}^{2/3} \cdot m_\mathrm{e}^{-2} = \frac{\hbar^2}{3 m_\mathrm{e}} n_\mathrm{e}^{5/3} \tag{12}$$

BUT: For typical WD centers ($\rho \sim 10^6\,\mathrm{g\,cm^{-3}}$), $v \sim 10^{10}\,\mathrm{cm\,s^{-1}} \to c$

In this case we need to calculate relativistically, and for $v = c$ find

$$P_\text{degen,rel.} = \frac{\hbar c}{3} n_\mathrm{e}^{4/3} \tag{13}$$

Are all white dwarfs stable?

The energy density of a plasma is

$$U = \begin{cases} \frac{3}{2}P & \text{non-relativistic} \\ 3P & \text{relativistic} \end{cases} \tag{14}$$

Therefore the total energy of a star/white dwarf is

$$E_{\text{tot}} = E_{\text{grav}} + E_{\text{gas}} = -\frac{GM^2}{R} + UV \tag{15}$$

Scaling relationships:

$$V \propto R^3 \quad \rho \propto R^{-3} \quad U \propto \rho^\Gamma \propto R^{-3\Gamma} \tag{16}$$

where

$$\Gamma = \begin{cases} 5/3 & \text{non-relativistic} \\ 4/3 & \text{relativistic} \end{cases} \tag{17}$$

For a star with non-relativistic gas:

$$E_{\text{tot}} = -\frac{A}{R} + \frac{B}{R^{3(\Gamma-1)}} = -\frac{A}{R} + \frac{B}{R^2} \tag{18}$$

This has a minimum at a finite radius

$\Longrightarrow$ star is stable

Are all white dwarfs stable?

The energy density of a plasma is

$$U = \begin{cases} \frac{3}{2}P & \text{non-relativistic} \\ 3P & \text{relativistic} \end{cases} \tag{19}$$

Therefore the total energy of a star/white dwarf is

$$E_{\text{tot}} = E_{\text{grav}} + E_{\text{gas}} = -\frac{GM^2}{R} + UV \tag{20}$$

Scaling relationships:

$$V \propto R^3 \quad \rho \propto R^{-3} \quad U \propto \rho^{\Gamma} \propto R^{-3\Gamma} \tag{21}$$

where

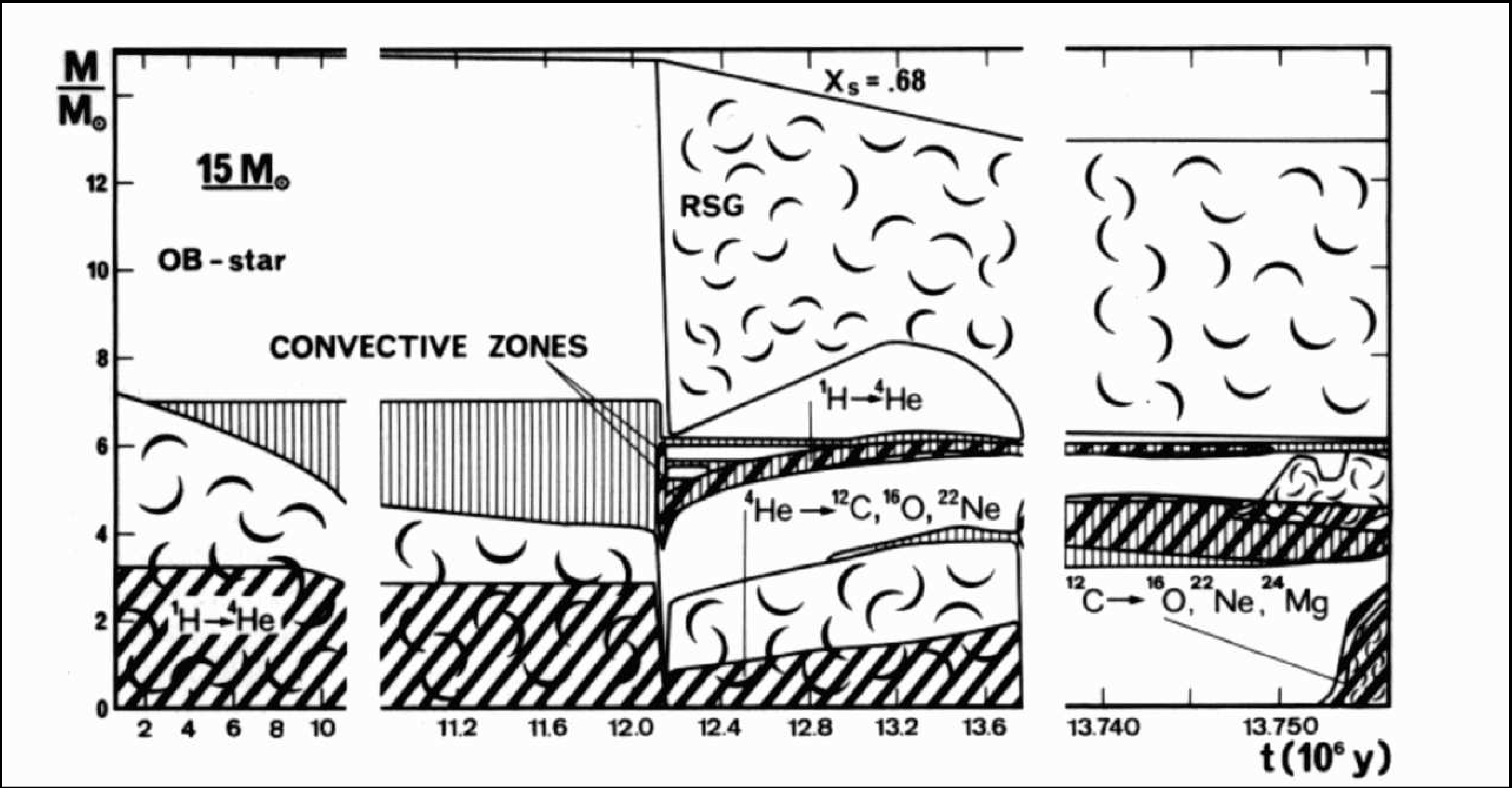
$$\Gamma = \begin{cases} 5/3 & \text{non-relativistic} \\ 4/3 & \text{relativistic} \end{cases} \tag{22}$$

For a star with relativistic gas:

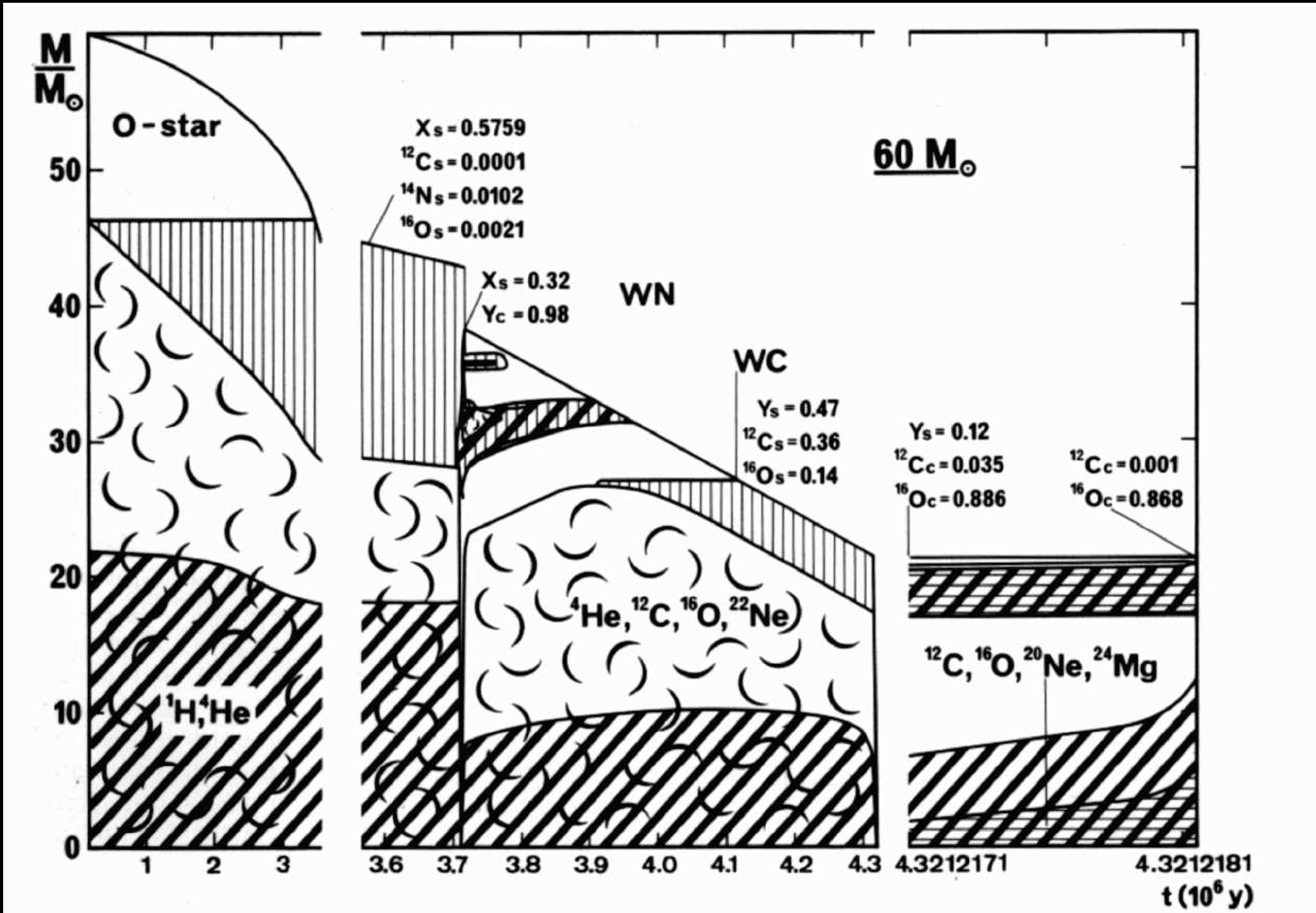
$$E_{\text{tot}} = -\frac{A}{R} + \frac{B}{R^{3(\Gamma-1)}} = -\frac{A}{R} + \frac{B}{R} \propto \frac{1}{R} \tag{23}$$

This has no minimum

$\Longrightarrow$ star is not stable $\Longrightarrow$ White Dwarfs have a maximum mass of $\sim 1.4 M_{\odot}$

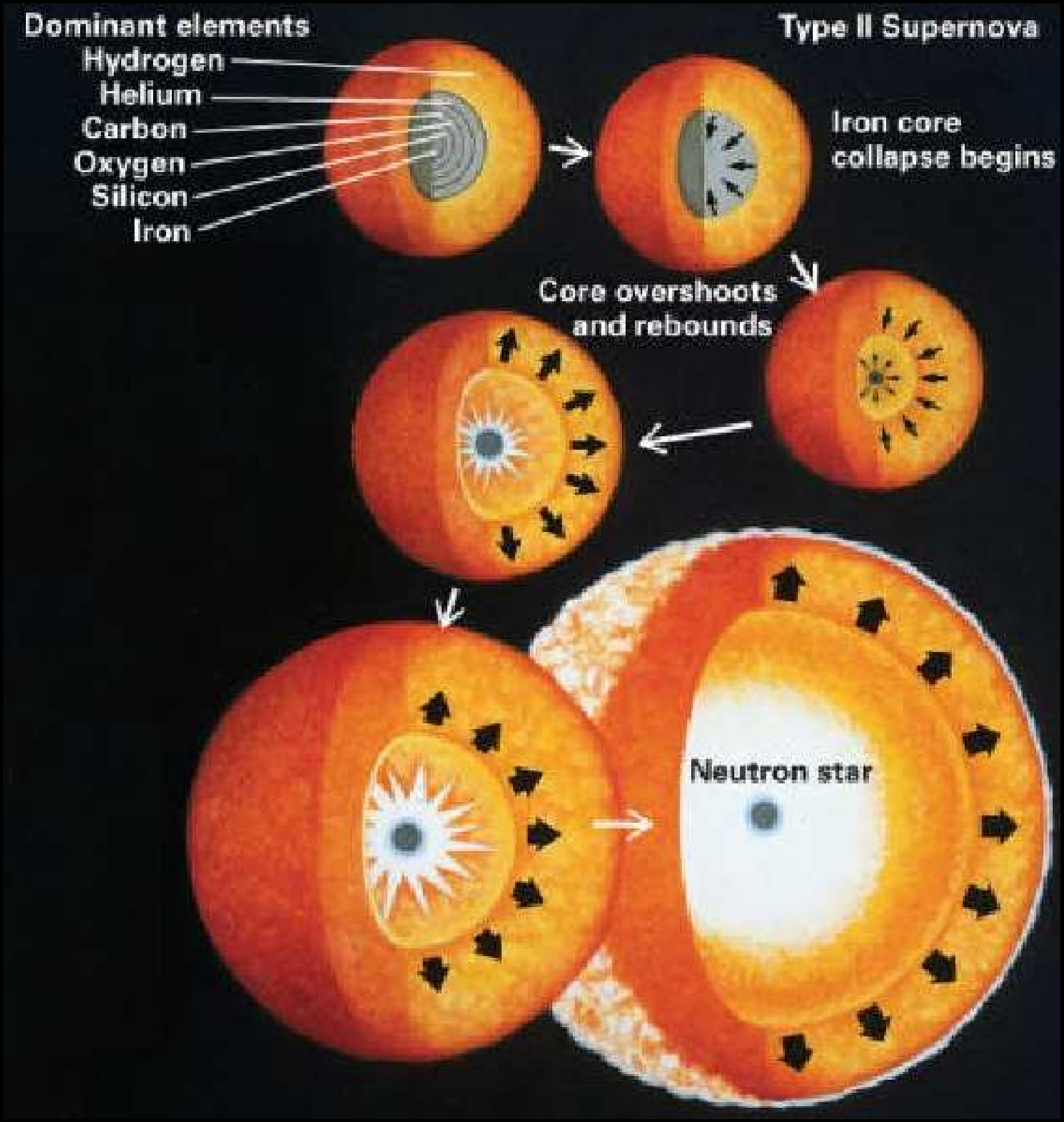


Evolution of the internal structure of a 15 $M_\odot$ star.



Evolution of the internal structure of a 60 $M_\odot$ star.

Note the very strong mass loss!



Massive stars: Fusion possible until ^{56}Fe

Photodesintegration sets in:

$$^{56}\mathrm{Fe} + \gamma \longrightarrow 13\,^{4}\mathrm{He} + 4\,\mathrm{n}$$
$$^{4}\mathrm{He} + \gamma \longrightarrow 2\,\mathrm{p} + 2\,\mathrm{n}$$

. . . and Neutronization:

$$\mathrm{p} + \mathrm{e}^{-} \longrightarrow \mathrm{n} + \nu_{\mathrm{e}}$$

$\Longrightarrow$ Core collapse supernova

$\Longrightarrow$ Neutron Star

Neutron stars are stable because pressure is dominated by neutrons, not electrons

$\Longrightarrow$ Neutrons have much larger mass than electrons, and therefore are non-relativistic

$\Longrightarrow$ stable configuration exists

Let's estimate the density:

De Broglie for relativistic particles ($p \sim mc$)

$$\lambda = \frac{h}{p} \sim \frac{h}{mc} \tag{24}$$

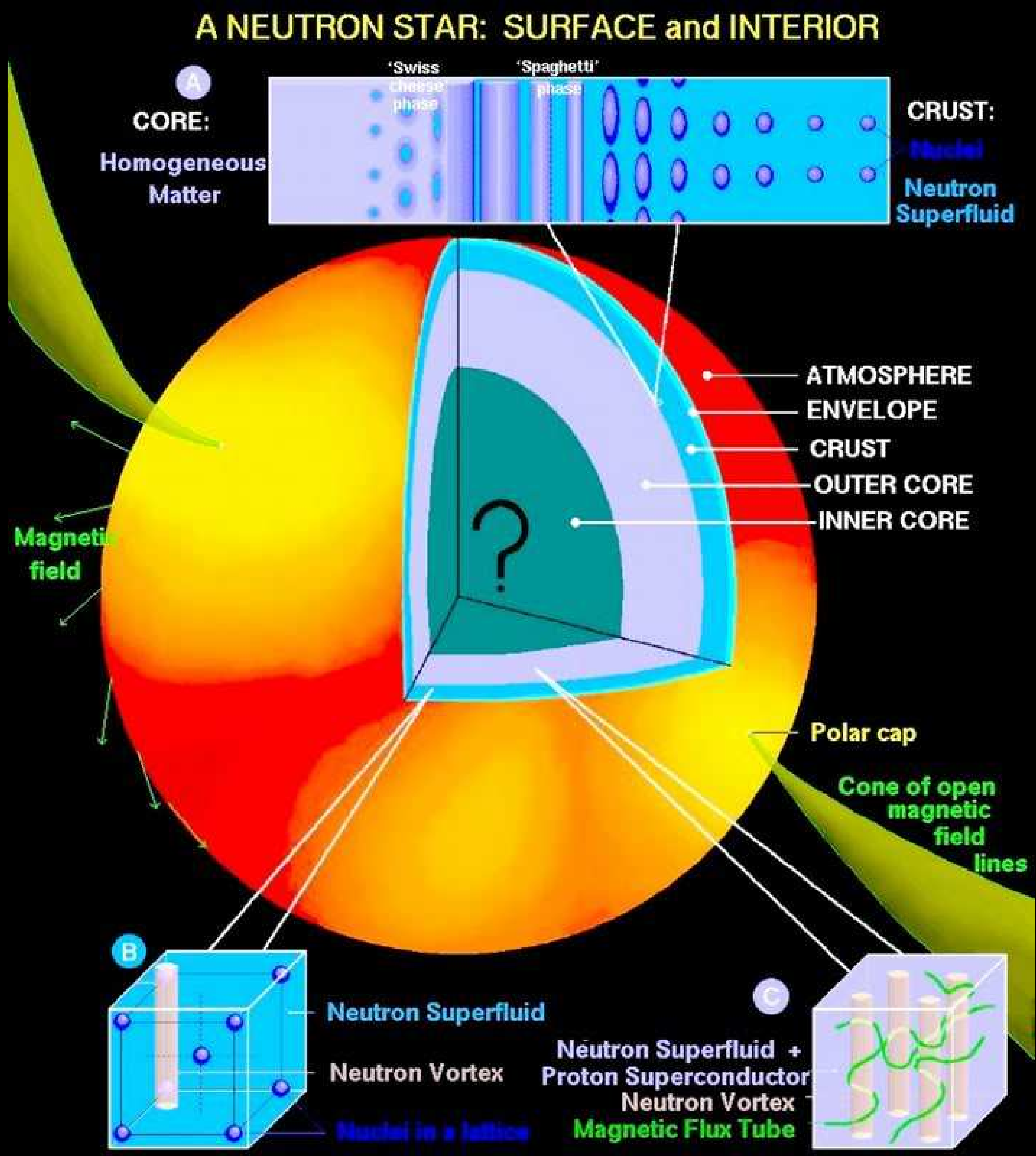
where λ is the Compton wavelength

Therefore for degenerate neutrons

$$\rho \sim \frac{m_\mathrm{n}}{\lambda_\mathrm{n}^3} \sim 7 \times 10^{14}\,\mathrm{g\,cm^{-3}} \tag{25}$$

$\Longrightarrow$ Nuclear densities

$\Longrightarrow$ Determination of precise equation of state is very difficult



The internal structure and other physical properties of neutron stars are virtually unknown!

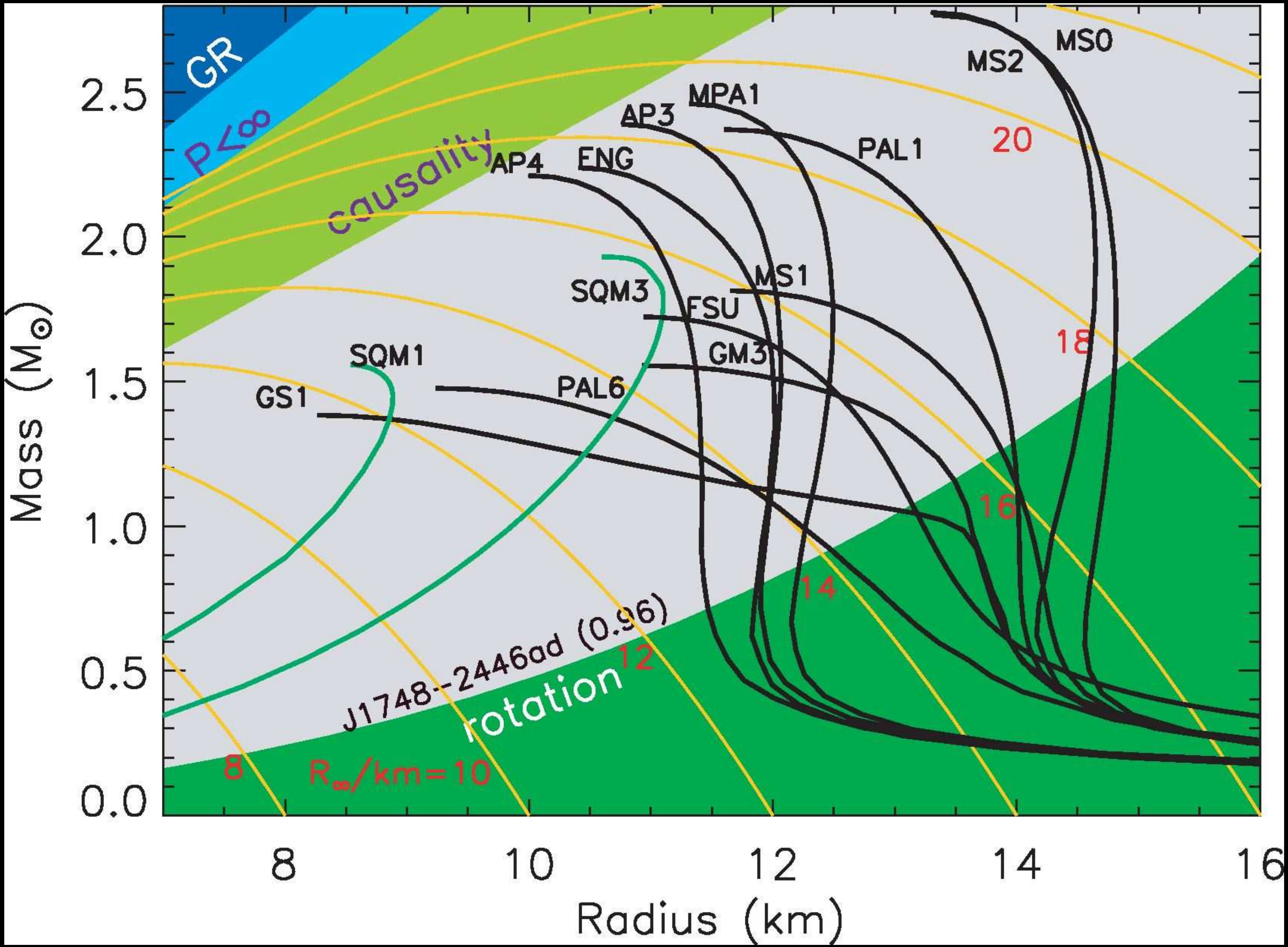
Some of the few facts known:

- typical mass $\sim$1.4 $M_\odot$
- strong magnetic fields ($>10^{12}$ G)

  for the cgs-challenged: 1 T $=$ 10^4 G

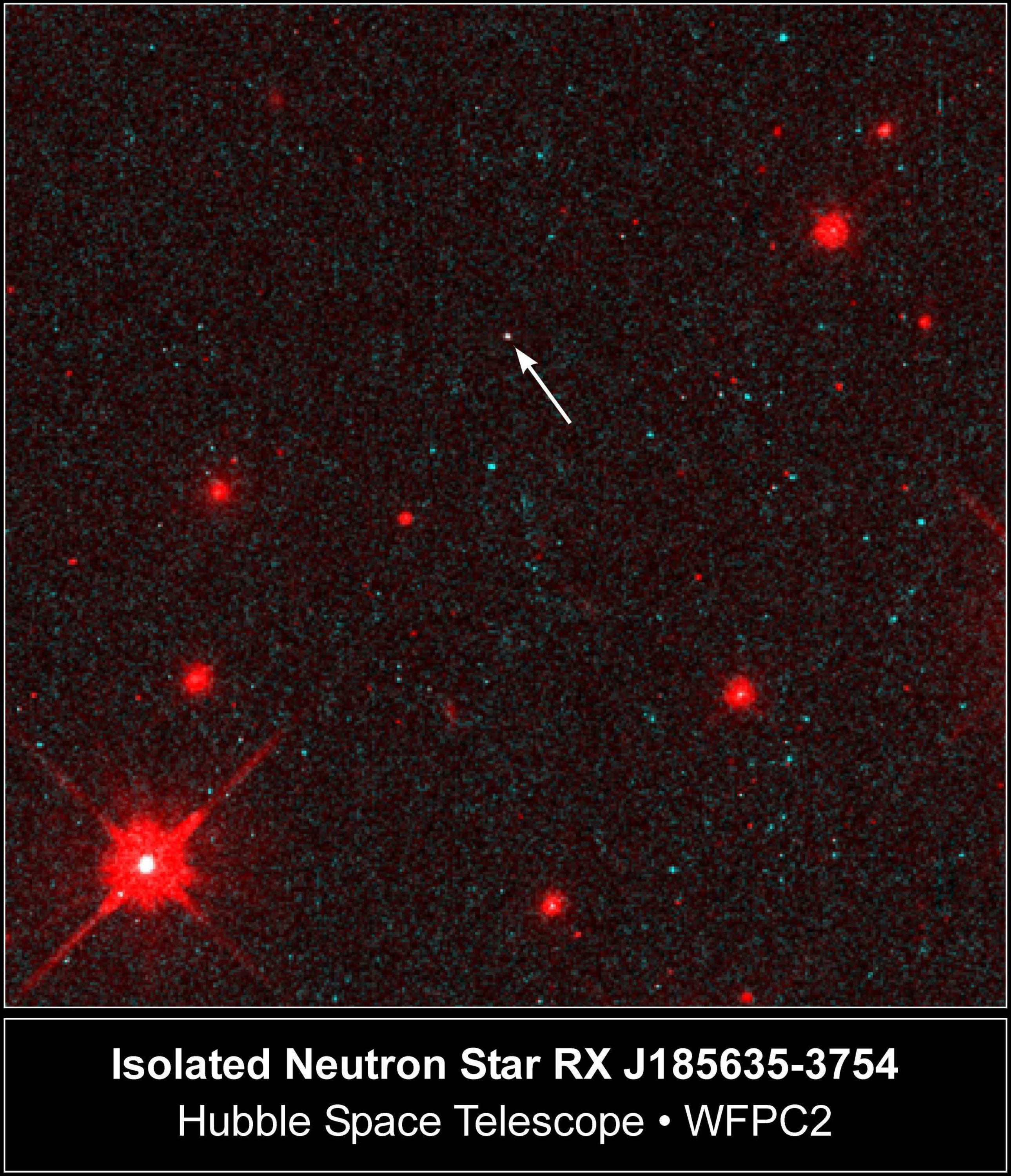
  $\Longrightarrow$ flux conservation during SN explosion
- fast rotation

  (P =msec to few 100 s)



Lattimer and Prakash (2001)

Mass-Radius-Relation for Neutron Stars depends strongly on EoS, i.e., measuring $M(R)$ would constrain nuclear equation of state.



Neutron stars are $\sim$10–15 km in radius

$\Longrightarrow$ Most isolated neutron stars are difficult to observe

$L = \sigma T^4 R^2$, $R_{\rm NS}/R_\odot = 10^{-5}$

Most knowledge of neutron stars comes from radio pulsars and neutron stars in X-ray binaries.

During supernova collapse, angular momentum is conserved (Explosion: symmetric)

Total angular momentum of homogeneous sphere:

$$J = I\omega \quad \text{where} \quad I = \frac{2}{5}MR^2 \tag{26}$$

During supernova collapse, angular momentum is conserved (Explosion: symmetric)

Total angular momentum of homogeneous sphere:

$$J = I\omega \quad \text{where} \quad I = \frac{2}{5}MR^2 \tag{27}$$

Angular momentum conservation ($J_{\text{before}} = J_{\text{NS}}$):

$$\frac{2}{5}M_{\text{before}}R_{\text{before}}^2\omega_{\text{before}} = \frac{2}{5}M_{\text{NS}}R_{\text{NS}}^2\omega_{\text{NS}} \tag{28}$$

or

$$\omega_{\text{NS}} = \left(\frac{M_{\text{before}}}{M_{\text{NS}}}\right)\left(\frac{R_{\text{before}}}{R_{\text{NS}}}\right)^2\omega_{\text{before}} \quad \text{or} \quad P_{\text{NS}} \sim \left(\frac{R_{\text{NS}}}{R_{\text{before}}}\right)^2 P_{\text{before}} \tag{29}$$

(where P: rotation period)

During supernova collapse, angular momentum is conserved (Explosion: symmetric)

Total angular momentum of homogeneous sphere:

$$J = I\omega \quad \text{where} \quad I = \frac{2}{5}MR^2 \tag{30}$$

Angular momentum conservation ($J_{\text{before}} = J_{\text{NS}}$):

$$\frac{2}{5}M_{\text{before}}R_{\text{before}}^2\omega_{\text{before}} = \frac{2}{5}M_{\text{NS}}R_{\text{NS}}^2\omega_{\text{NS}} \tag{31}$$

or

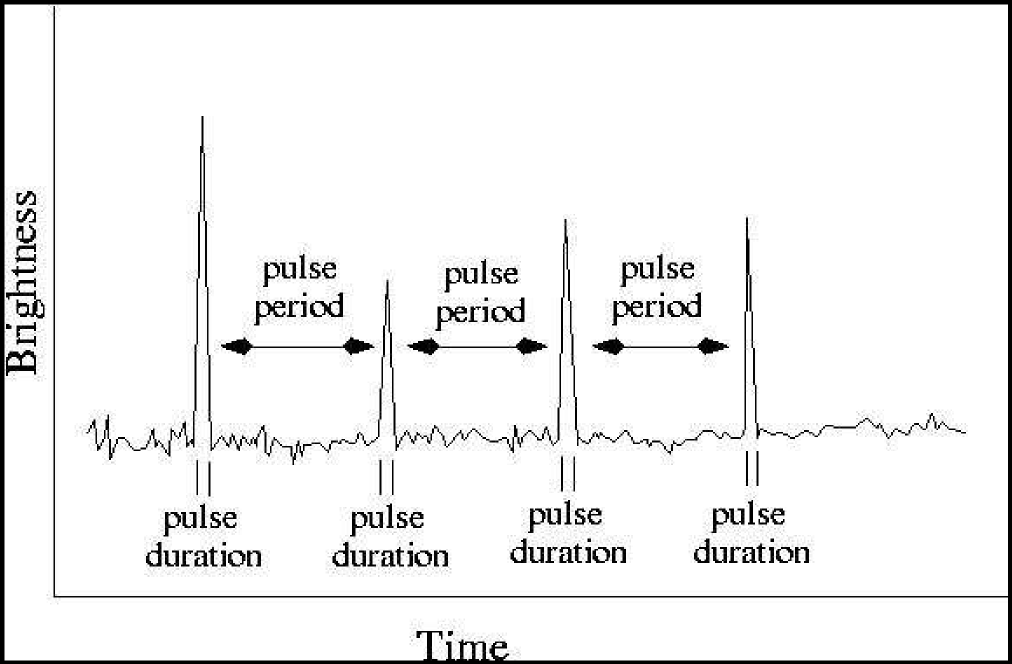
$$\omega_{\text{NS}} = \left(\frac{M_{\text{before}}}{M_{\text{NS}}}\right)\left(\frac{R_{\text{before}}}{R_{\text{NS}}}\right)^2\omega_{\text{before}} \quad \text{or} \quad P_{\text{NS}} \sim \left(\frac{R_{\text{NS}}}{R_{\text{before}}}\right)^2 P_{\text{before}} \tag{32}$$

(where P: rotation period)

Example: $R_{\text{before}} = 700000\,\text{km}$ (sun), $R_{\text{NS}} = 15\,\text{km}$, $P_{\text{Sun}} = 27\,\text{d} \Longrightarrow P_{\text{NS}} = 1\,\text{ms}$

Neutron Stars are extremely fast rotators.

close to break-up speed!



Discovery: Bell & Hewish (1967):

Radio Pulsar

radio emission is pulsed,

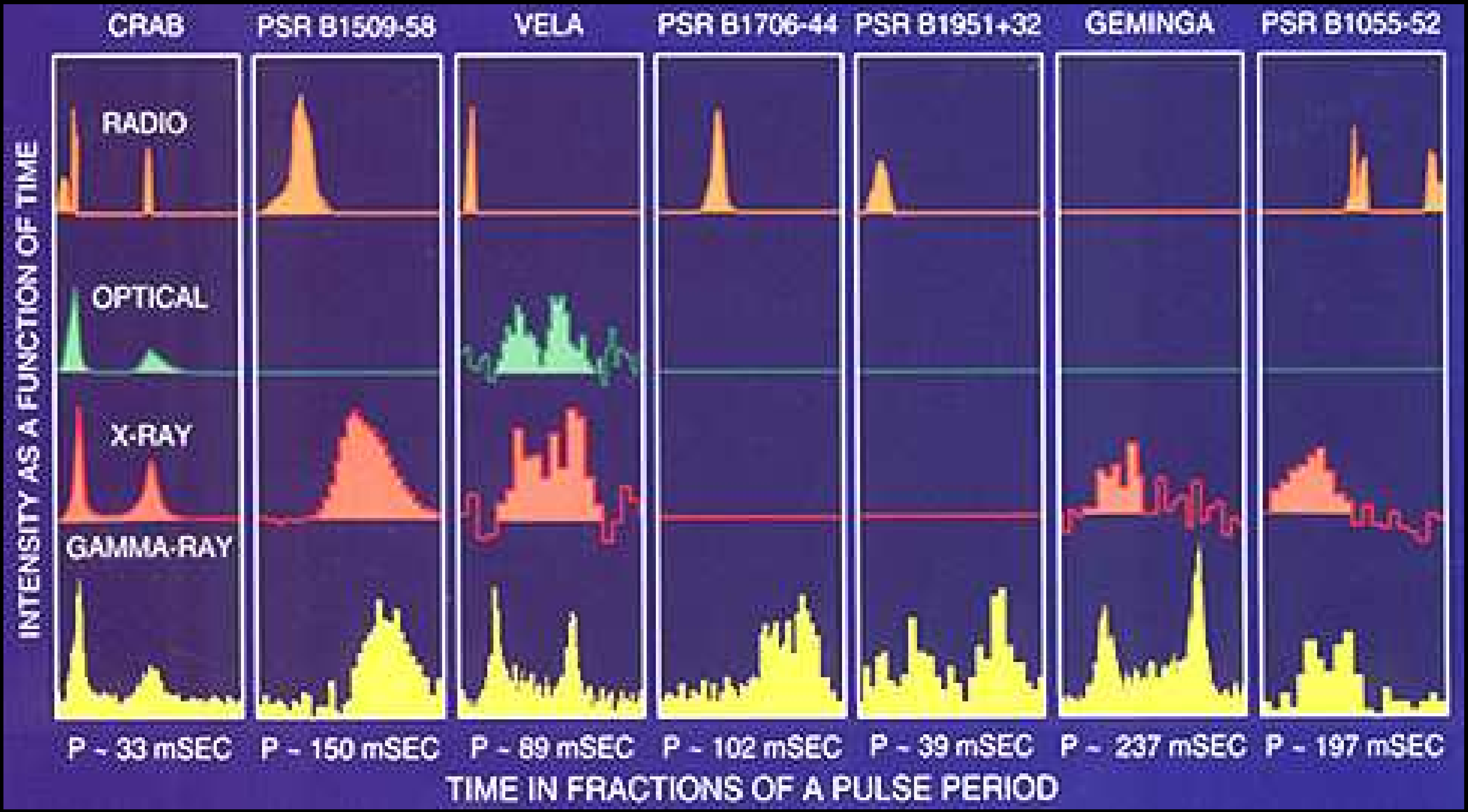
very short periods: milliseconds to a few seconds

Sounds:

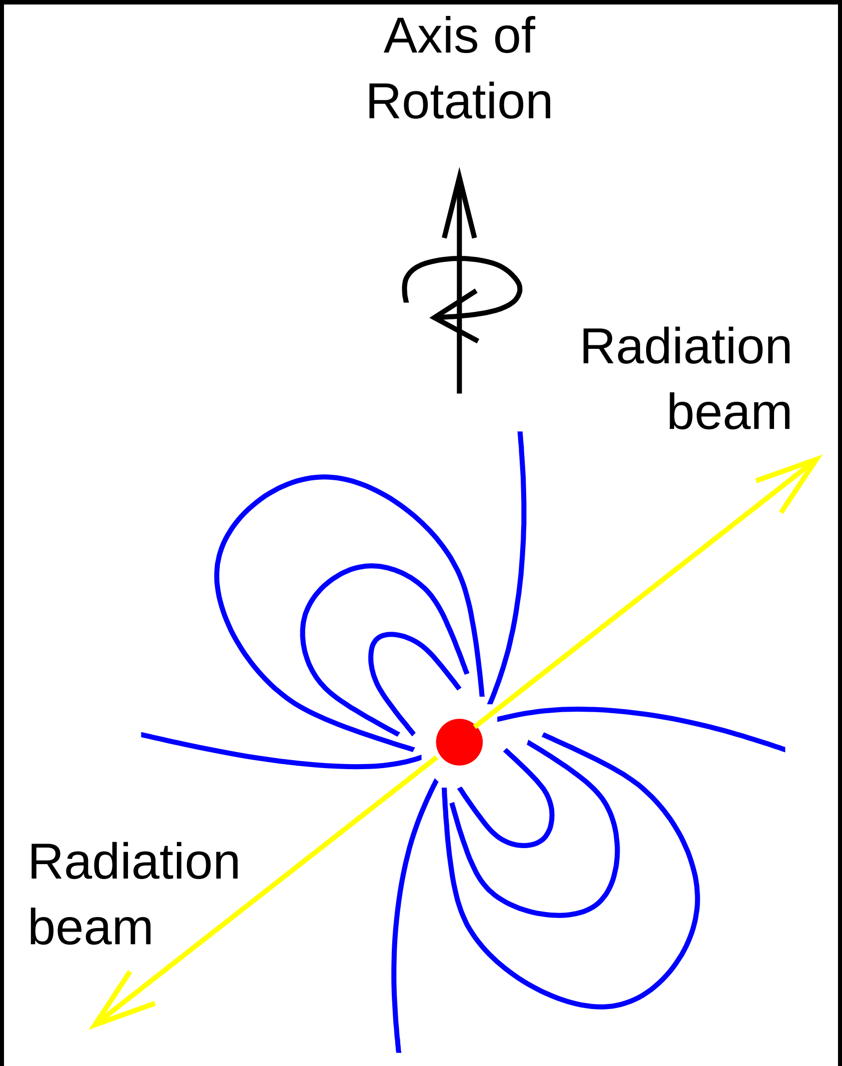
- PSR 0329 – a normal pulsar ($P = 0.714519$ s)
- PSR 0833 – the Vela pulsar, a faster, younger pulsar in the Vela supernova remnant ($P = 89$ msec)
- Crab pulsar – the youngest pulsar ($P = 33$ ms)
- B1937 – one of the fastest pulsars ($P = 0.00155780644887275$ s)

See/hear `http://www.jb.man.ac.uk/~pulsar/Education/Sounds/sounds.html` for more examples.

Pulsars at different wavelengths



Pulsations not only in the radio regime, but also at optical, X-ray, and γ-ray wavelengths (but not in all cases)



"Lighthouse model" for pulsars

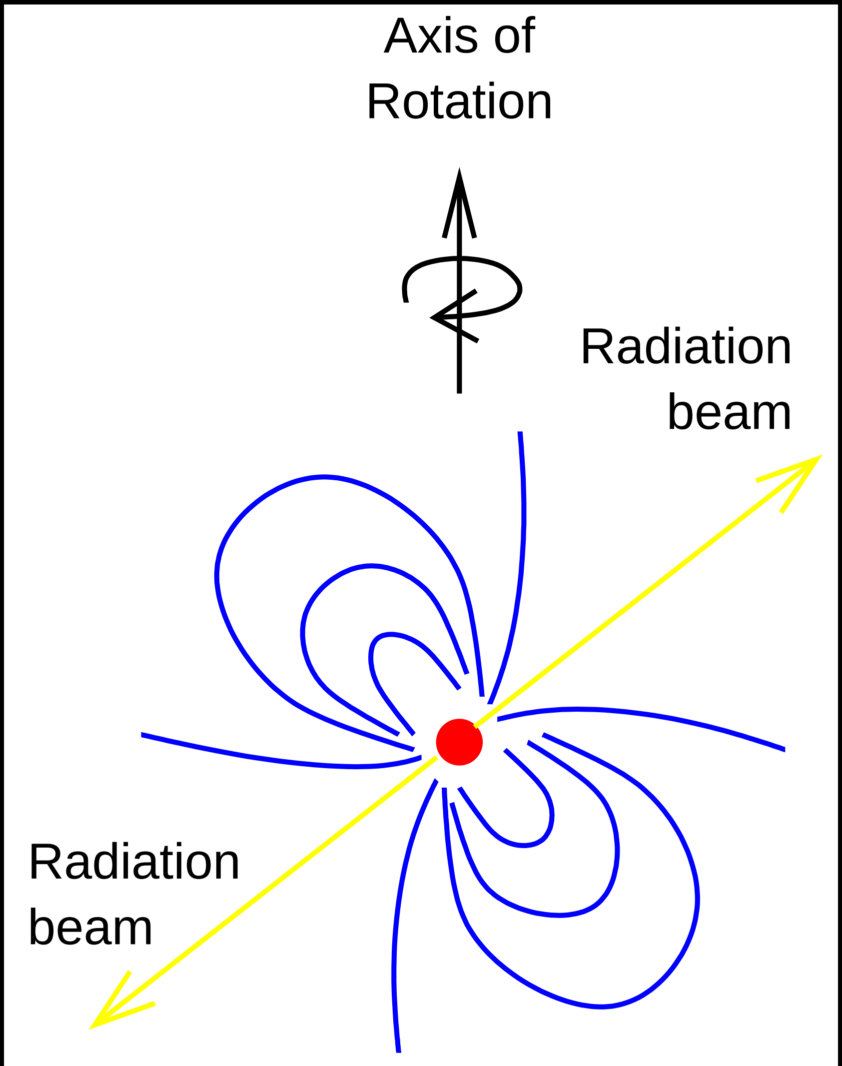
Another conserved quantity:
magnetic flux: $\Phi = BR^2$

Magnetic field after SN:

$$B_{\text{NS}} = \left(\frac{R_{\text{before}}}{R_{\text{NS}}}\right)^2 B_{\text{before}}$$

$\Longrightarrow$ neutron stars have strong magnetic fields (typical: $B \sim 10^6 \dots 10^8$ T)

Radio pulsars are fast rotating (isolated) neutron stars with strong magnetic fields.



“Lighthouse model” for pulsars

Radio-emission is related to magnetic field of neutron star.
For a dipole:

$$B(r) = B_{\mathrm{s}} \left(\frac{r}{R_{\mathrm{s}}}\right)^{-3} \quad (33)$$

Because of rotation, linear velocity of B-field reaches c at the light cylinder

$$R_{\mathrm{L}} = \frac{c}{\omega} = \frac{c}{2\pi} P \quad (34)$$

at a few 1000 km from object

The rotating B-field must radiate away energy in order not to violate causality

Amount of energy in light cylinder:

$$U \sim \frac{B^2}{8\pi} = \frac{1}{8\pi}\left(B_\mathsf{s}\left(\frac{R_\mathsf{L}}{R_\mathsf{s}}\right)^{-3}\right)^2 = \frac{B_\mathsf{s}^2}{8\pi}\frac{R_\mathsf{s}^6}{R_\mathsf{L}^6} \tag{35}$$

Energy flux radiated away per unit area is given by Poynting vector, $S = Uc$

Therefore loss of energy (remember $R_\mathsf{L} = c/\omega$):

$$\dot{E} = -4\pi R_\mathsf{L} \cdot U = -\frac{1}{2}\frac{B_\mathsf{s}^2 R_\mathsf{s}^6}{R_\mathsf{L}^4}c = -\frac{1}{2}B_\mathsf{s}^2 R_\mathsf{s}^6 \omega^4\, c^{-3} \tag{36}$$

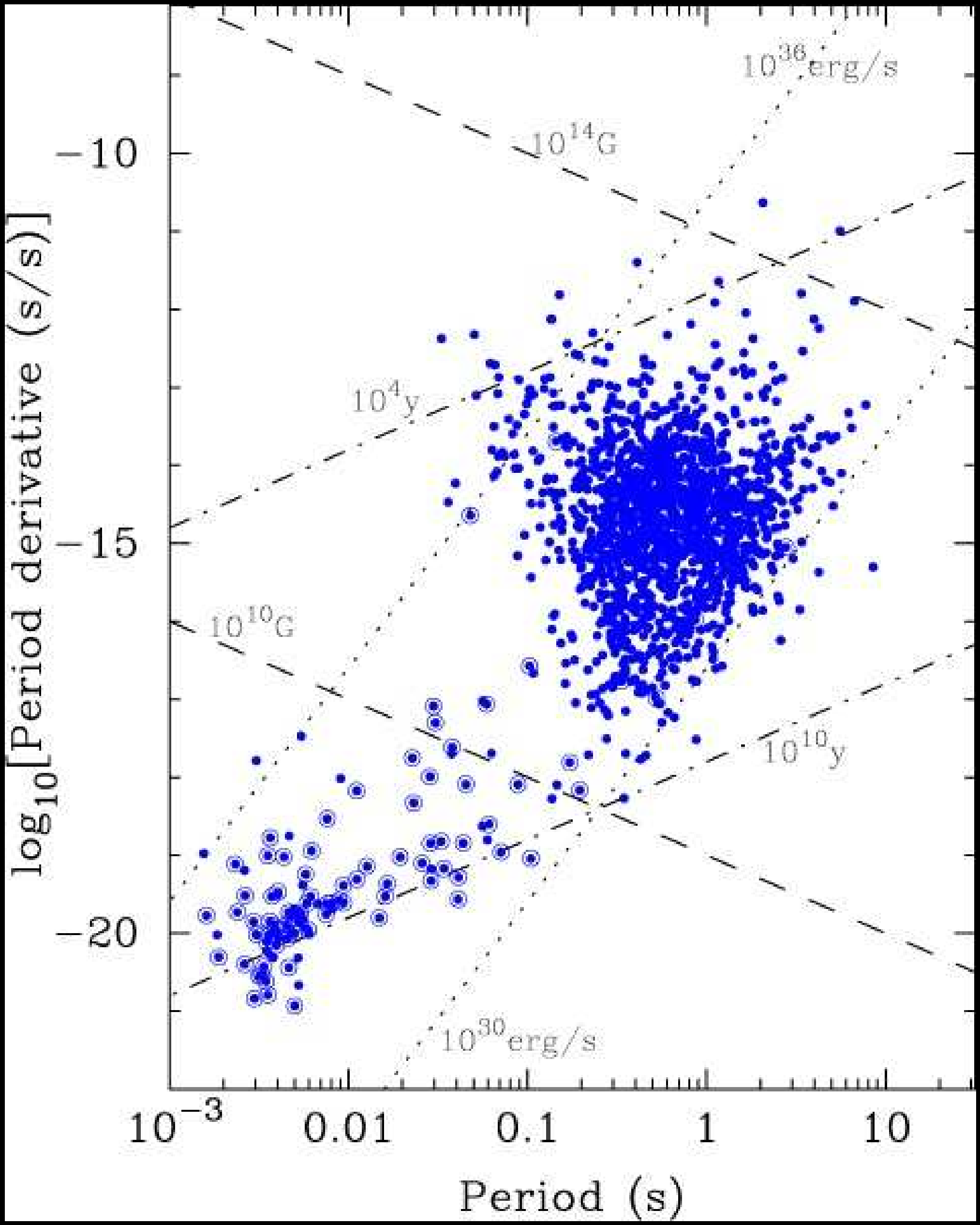
This energy comes from slow down of rotation of neutron star:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{I\omega^2}{2}\right) = I\omega\dot{\omega} = -\frac{B_\mathsf{s}^2 R_\mathsf{s}^6 \omega^4}{2c^3} \tag{37}$$

This means that by measuring ω, $\dot{\omega}$ (or the pulse period P and its rate of change, $\dot{P}$), we can measure B:

$$B = -\frac{I\dot{\omega}c^3}{R_\mathsf{s}\omega^3} = \frac{I}{4\pi^2}\frac{P\dot{P}c^3}{R_\mathsf{s}^6} \tag{38}$$

Typical values found are $10^{11} < B < 10^{13}$ G.

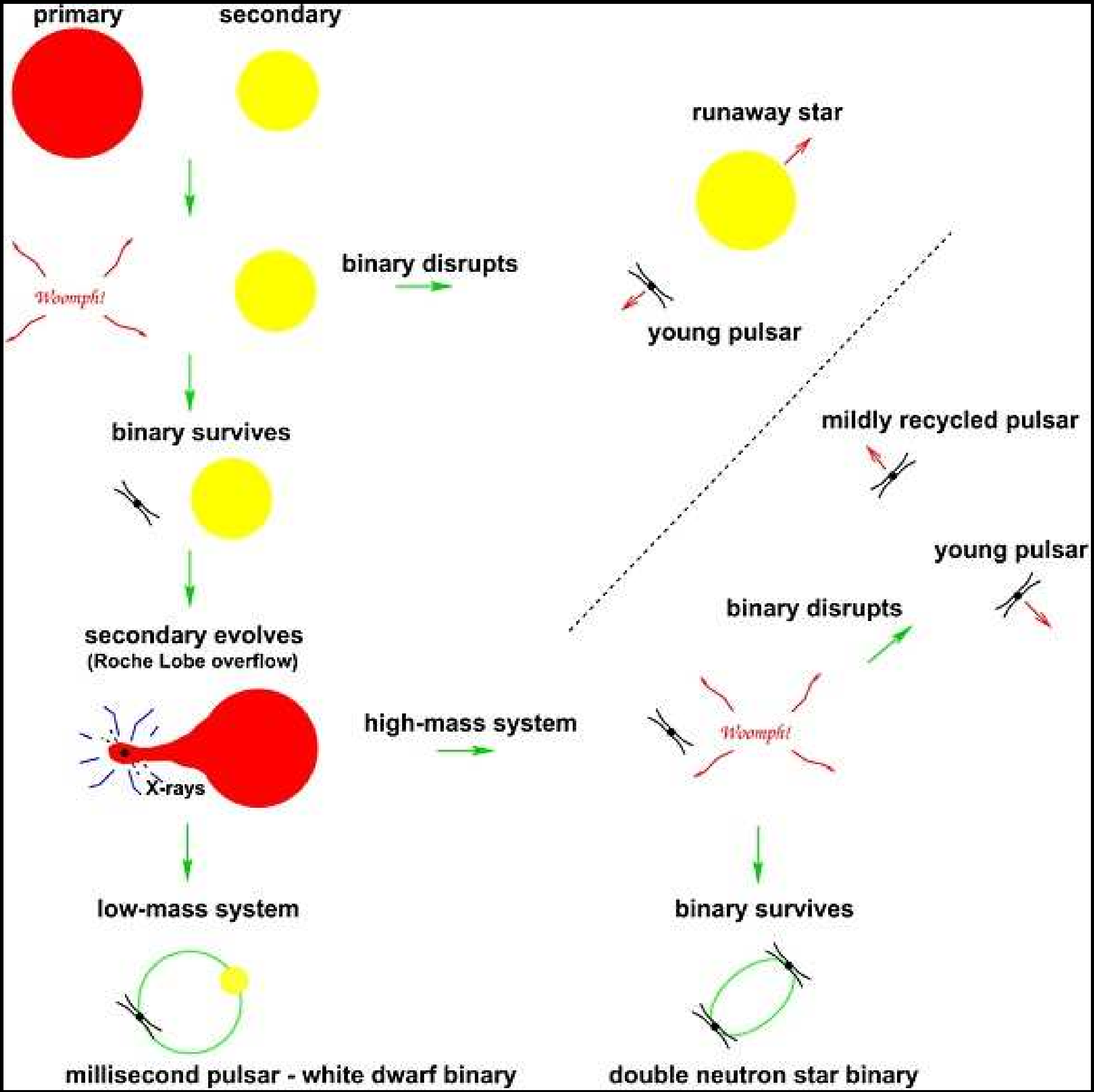


Pulsars show characteristic distribution in P-$\dot{P}$-diagram

Gives indication on age evolution:

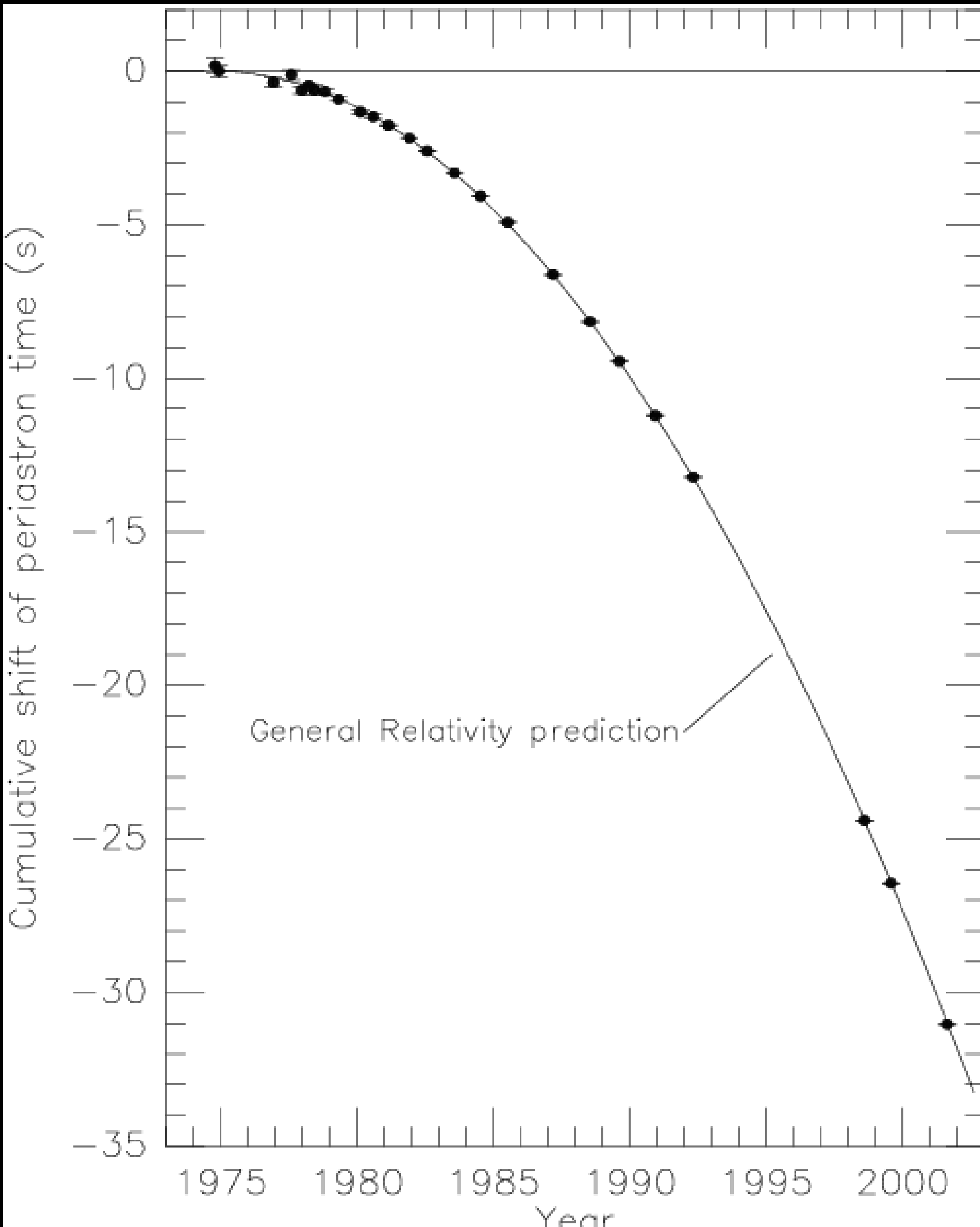
- Born with high B-field and high P
- Slowdown with $\sim$ constant P
- B-field decay?
- cross death line
- pulsars in binary systems may be reborn as millisecond pulsars.

Lorimer, Liv. Rev. Rel.



Lorimer

Some pulsars are born in binary systems, some of these evolve to binary pulsars



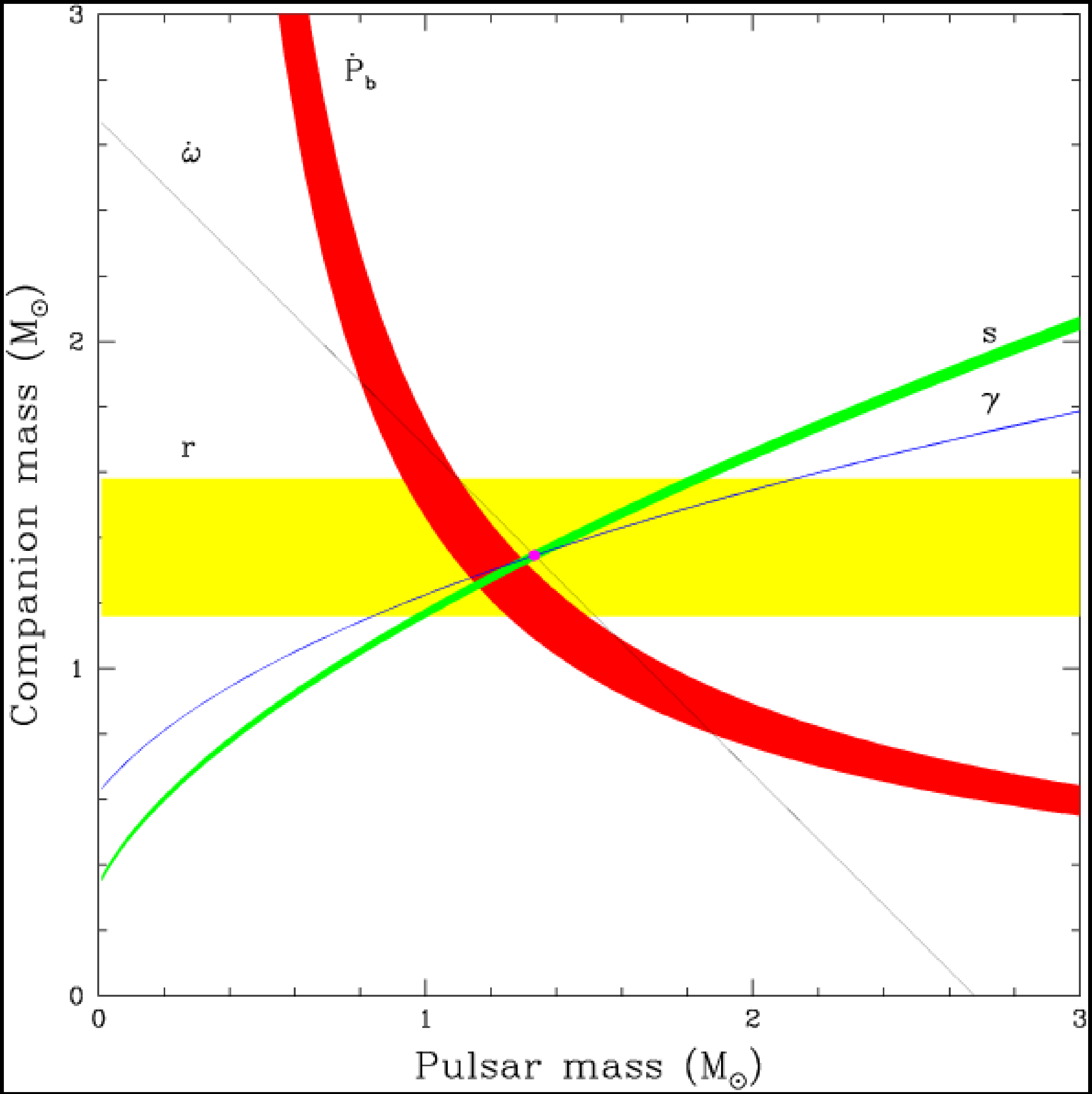
Pulsars are very precise clocks

$\Longrightarrow$ Can measure orbital position of neutron star using pulse arrival time measurements.

Slight delays in arrival times are due to relativistic effects, mainly due to emission of gravitational waves.

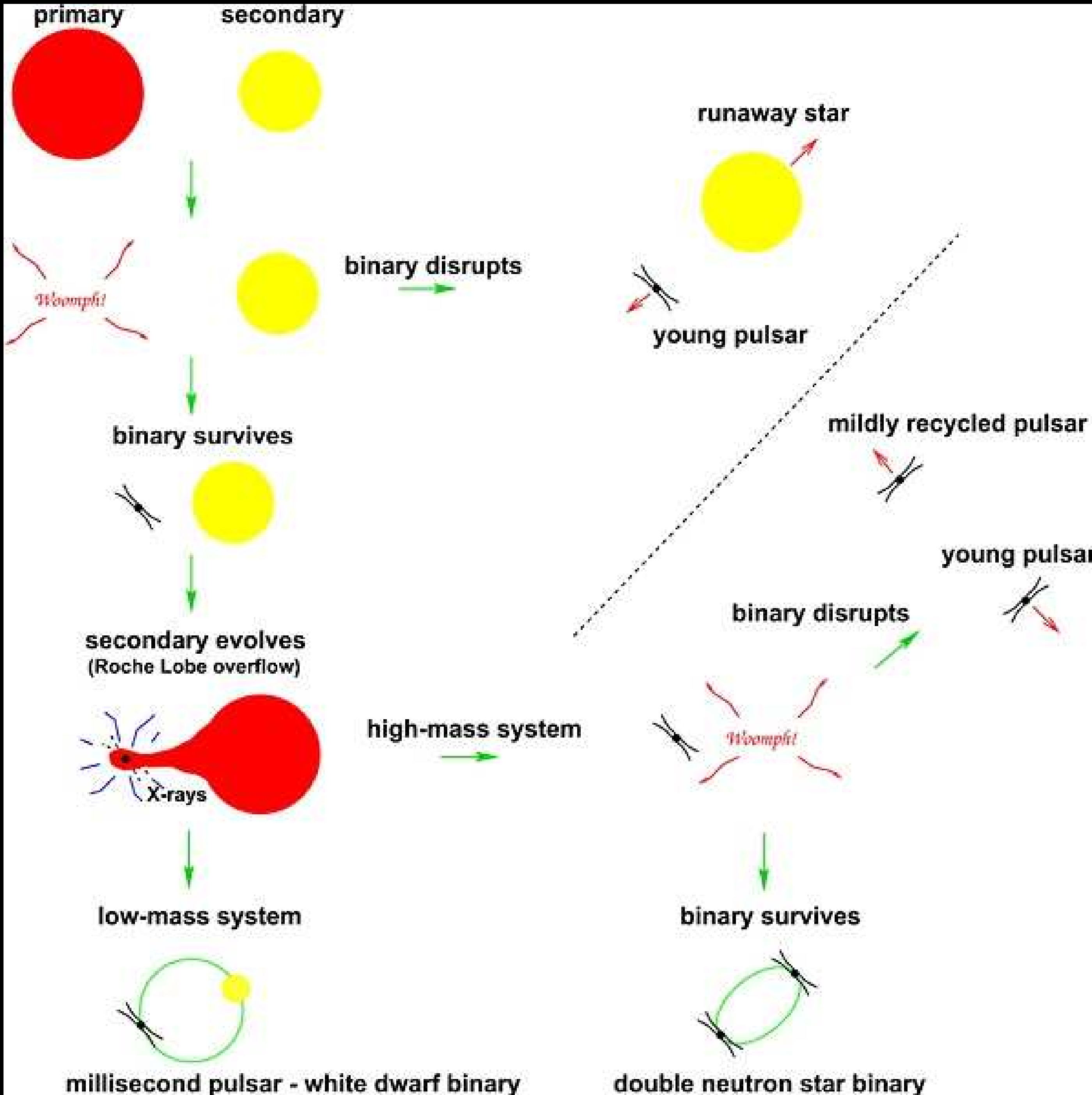
Note: this is still a measurement in the weak field limit!

I. Stairs

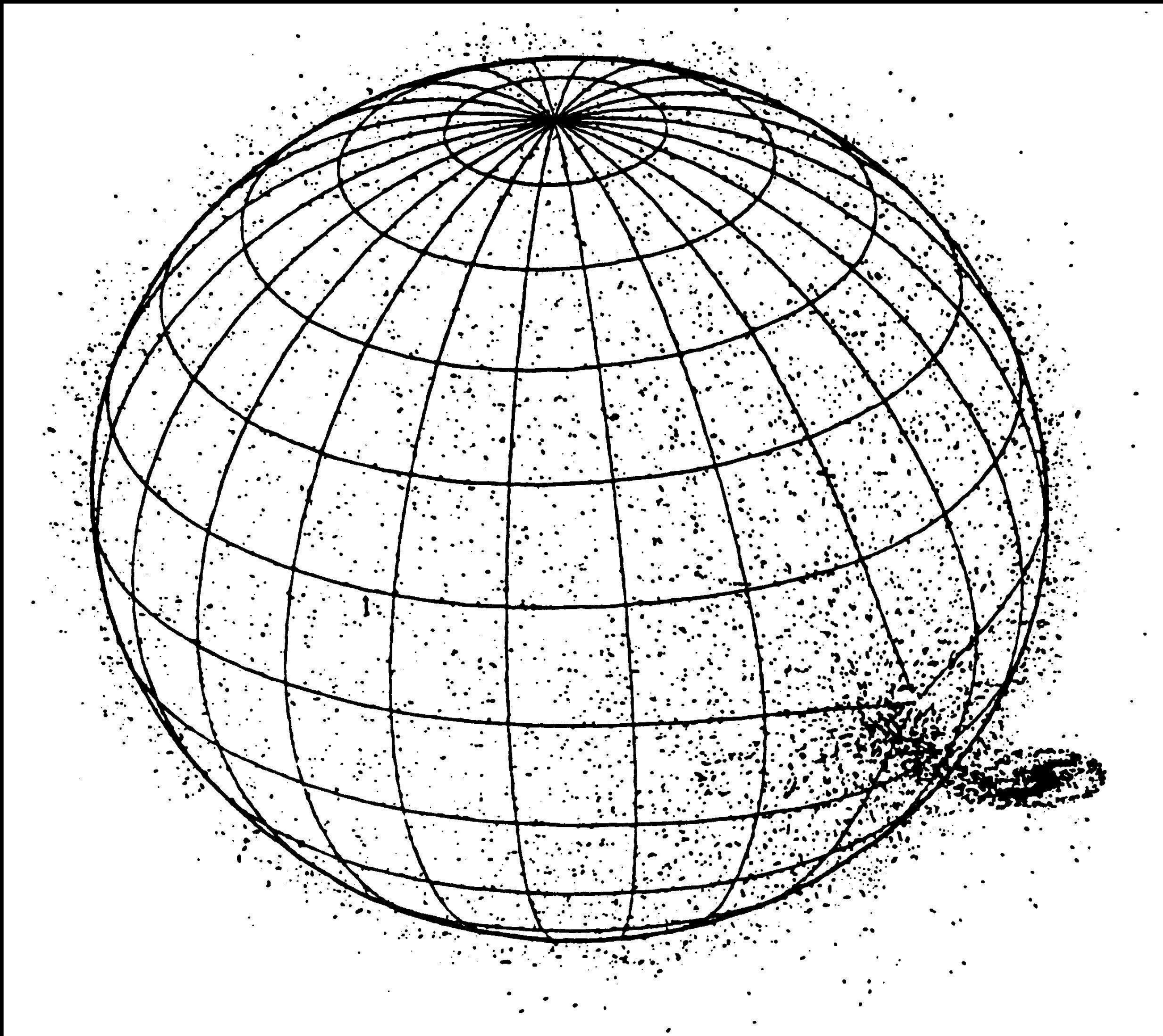


I. Stairs

Binary pulsars allow precise measurement of neutron star masses


primary
secondary
runaway star
Woomph!
binary disrupts
young pulsar
binary survives
mildly recycled pulsar
young pulsar
secondary evolves
(Roche Lobe overflow)
binary disrupts
high-mass system
Woomph!
X-rays
low-mass system
binary survives
millisecond pulsar - white dwarf binary
double neutron star binary

Lorimer

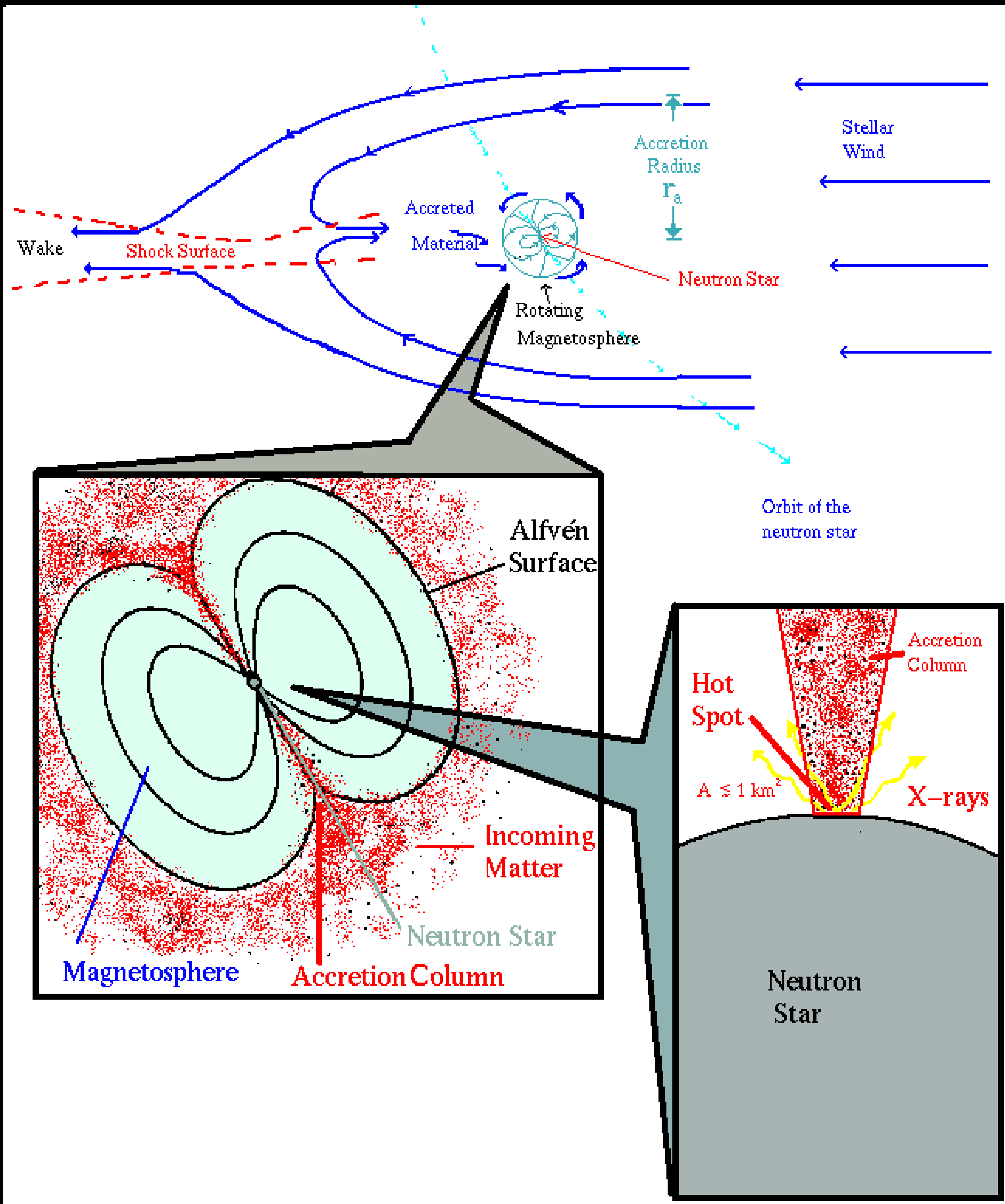


(SMC X-1; Dennerl, Dissertation MPE)

X-ray binary: neutron star accretes mass from donor star

- Low Mass X-ray Binaries (LMXB): donor late type $\Longrightarrow$ mainly old systems

  low B-fields, X-ray bursts, Quasi-Periodic Oscillations $\Longrightarrow$ very interesting, but unfortunately cannot discussed here for time reasons.
- High Mass X-ray Binaries (HMXB): donor early type $\Longrightarrow$ mainly young systems



Accreting plasma couples to B-field at Alfvén radius

$$r_{\text{mag}} = \left(\frac{8\pi^2}{G}\right)^{1/7} \left(\frac{R_\star^{12} B_{\text{p}}^4}{M \dot{M}^2}\right)^{1/7}$$

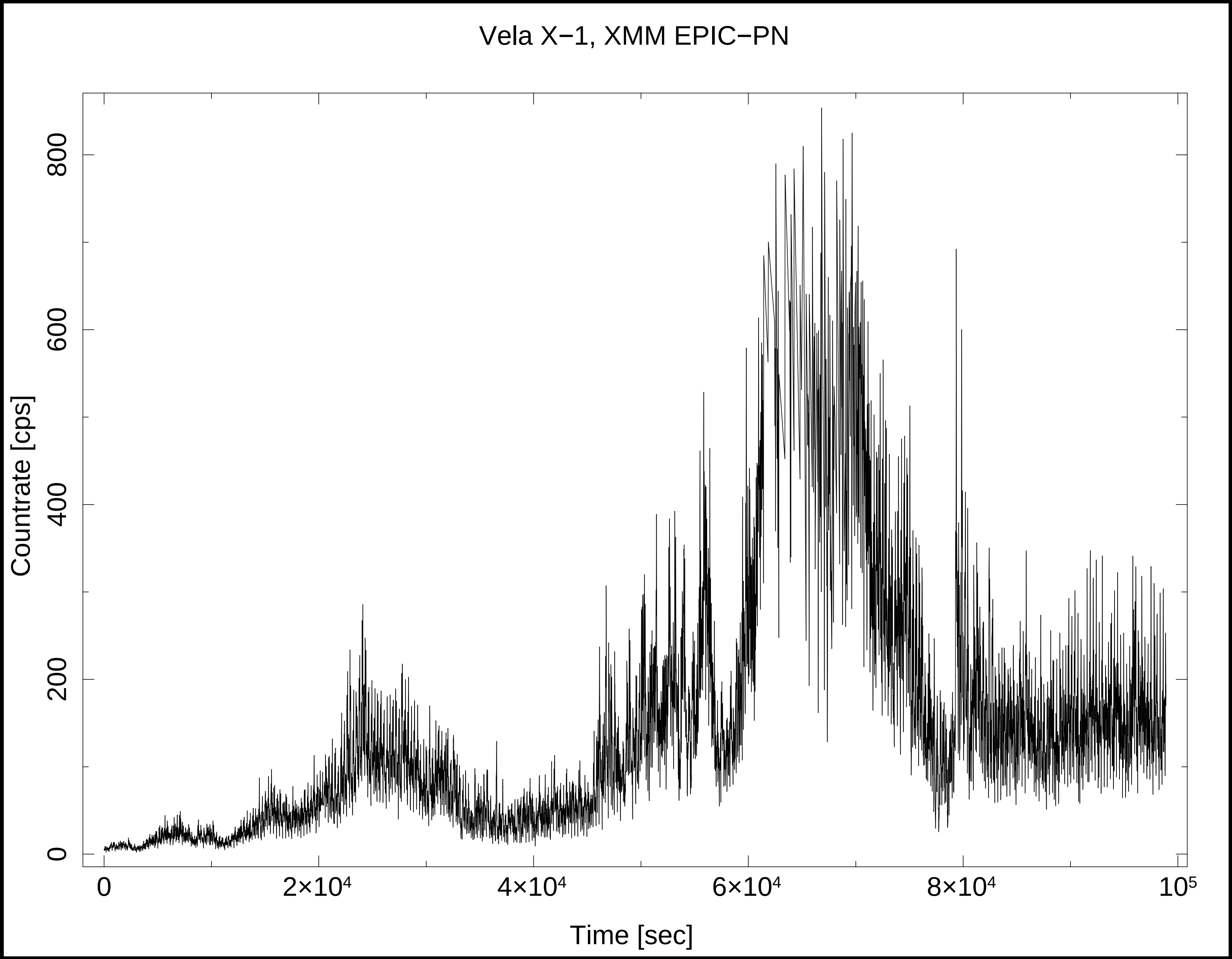
For typical neutron star parameters (1.44 $M_\odot$, $B \sim 10^{12}$ G):
$r_{\text{mag}} \sim 1800\,\text{km}$.

Typical parameters of accretion column:

- $\dot{M} \sim 10^{-9\ldots-11}\,\text{M}_\odot\,\text{yr}^{-1}$
- $v \sim 0.7\,c$

Useless number of the day:
$10^{-9}\,M_\odot\,\text{yr}^{-1} \sim 6 \times 10^{13}\,\text{kg}\,\text{s}^{-1}$,
or 1 Lake Erie every 8 sec.

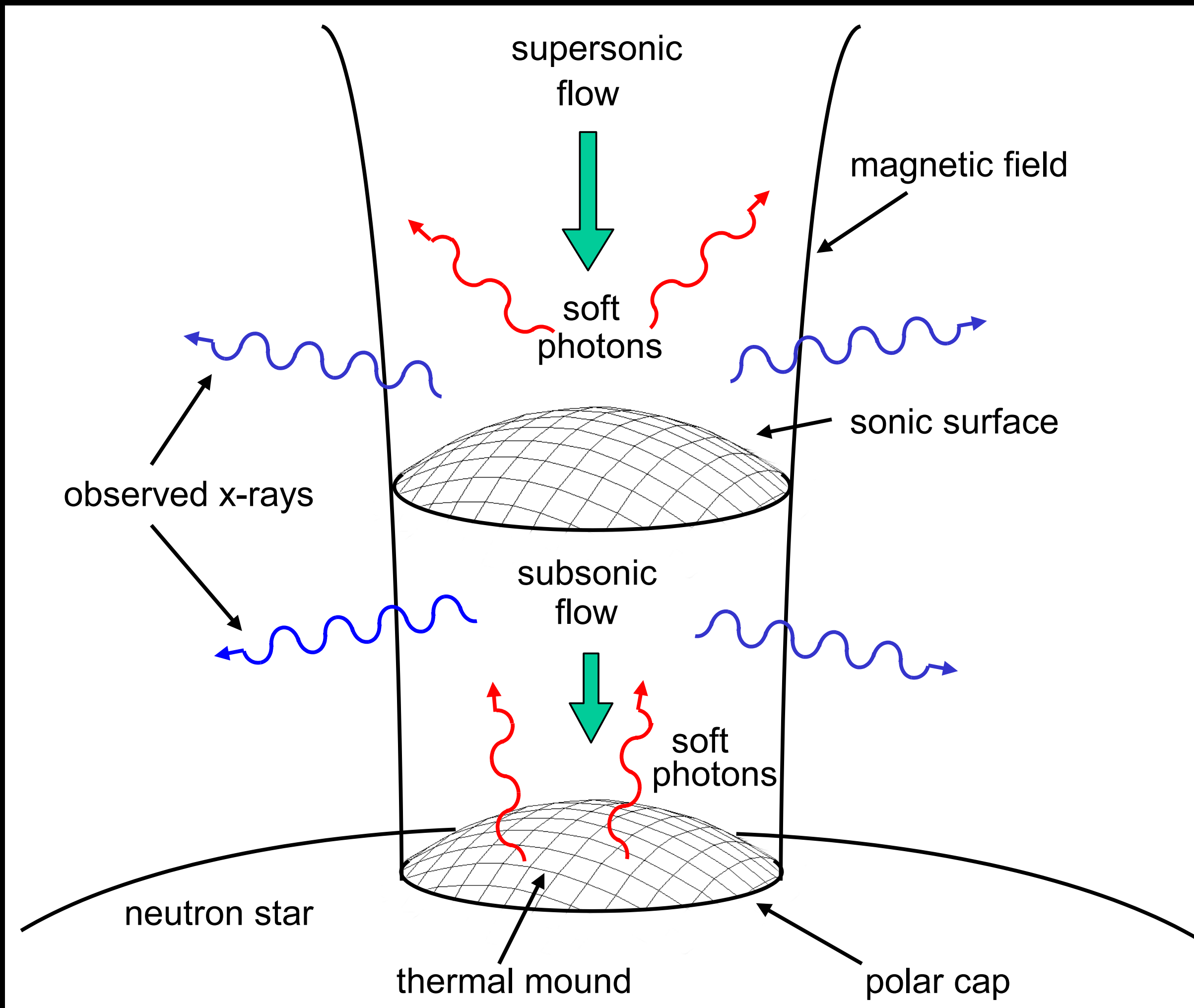
I. Negueruela
(after Davidson & Ostriker, 1973)



Vela X-1 (Fürst et al., in prep)

Accretion process can be very violent

$\Longrightarrow$ strong short term variations of $\dot{M}$

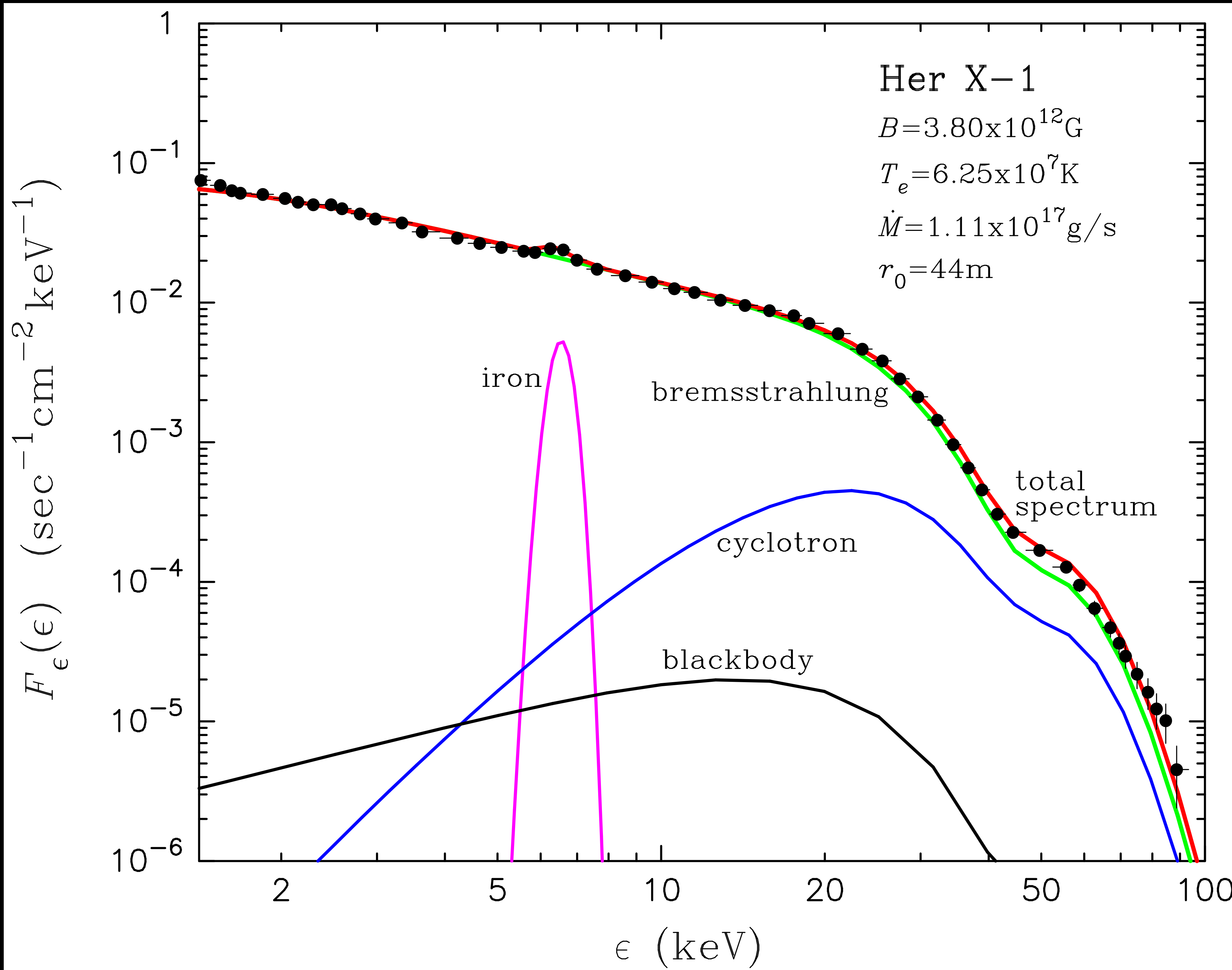


For high luminosity systems (=high mass accretion rate): Radiative shock dominates formation of observed contin-uum.

For low luminosity systems: accretion flow stopped by Coulomb interactions. Even less well understood.

Physics:

- accretion mound produces soft X-rays (bremsstrahlung)
- X-rays are upscattered in accretion shock (bulk motion Comptonization)
- hard X-rays diffuse through walls of accretion column


Her X−1
B=3.80x10^{12}G
T_e=6.25x10^{7}K
$\dot{M}$=1.11x10^{17}g/s
r_0=44m
iron
bremsstrahlung
total spectrum
cyclotron
blackbody
$F_\epsilon(\epsilon)$ (sec^{-1}cm^{-2}keV^{-1})
ϵ (keV)
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
2
5
10
20
50
100

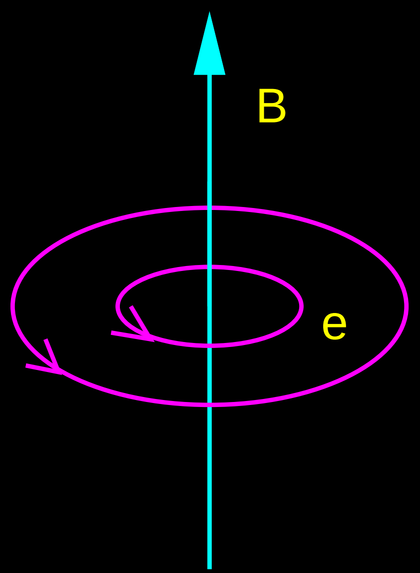
?, Fig. 6

Strong field at NS poles: Quantization of electron energies $\perp$ B-field lines (Landau levels):

$$E_n = m_{\mathrm{e}}c^2 \frac{\sqrt{1 + 2n(B/B_{\mathrm{crit}})\sin^2\theta} - 1}{\sin^2\theta}$$

$p_{\parallel}$: momentum of electron $\parallel$ B-field, n: major quantum number, B_{crit} is

$$B_{\mathrm{crit}} = \frac{m_{\mathrm{e}}^2 c^3}{\mathrm{e}\hbar} \sim 4.4 \times 10^{13}\,\mathrm{G}$$



For $B \ll B_{\mathrm{crit}}$, distance between Landau levels:

$$E_{\mathrm{cyc}} = \frac{\hbar e}{m_{\mathrm{e}}c}B = 11.6\,\mathrm{keV}\left(\frac{B}{10^{12}\,\mathrm{G}}\right)$$

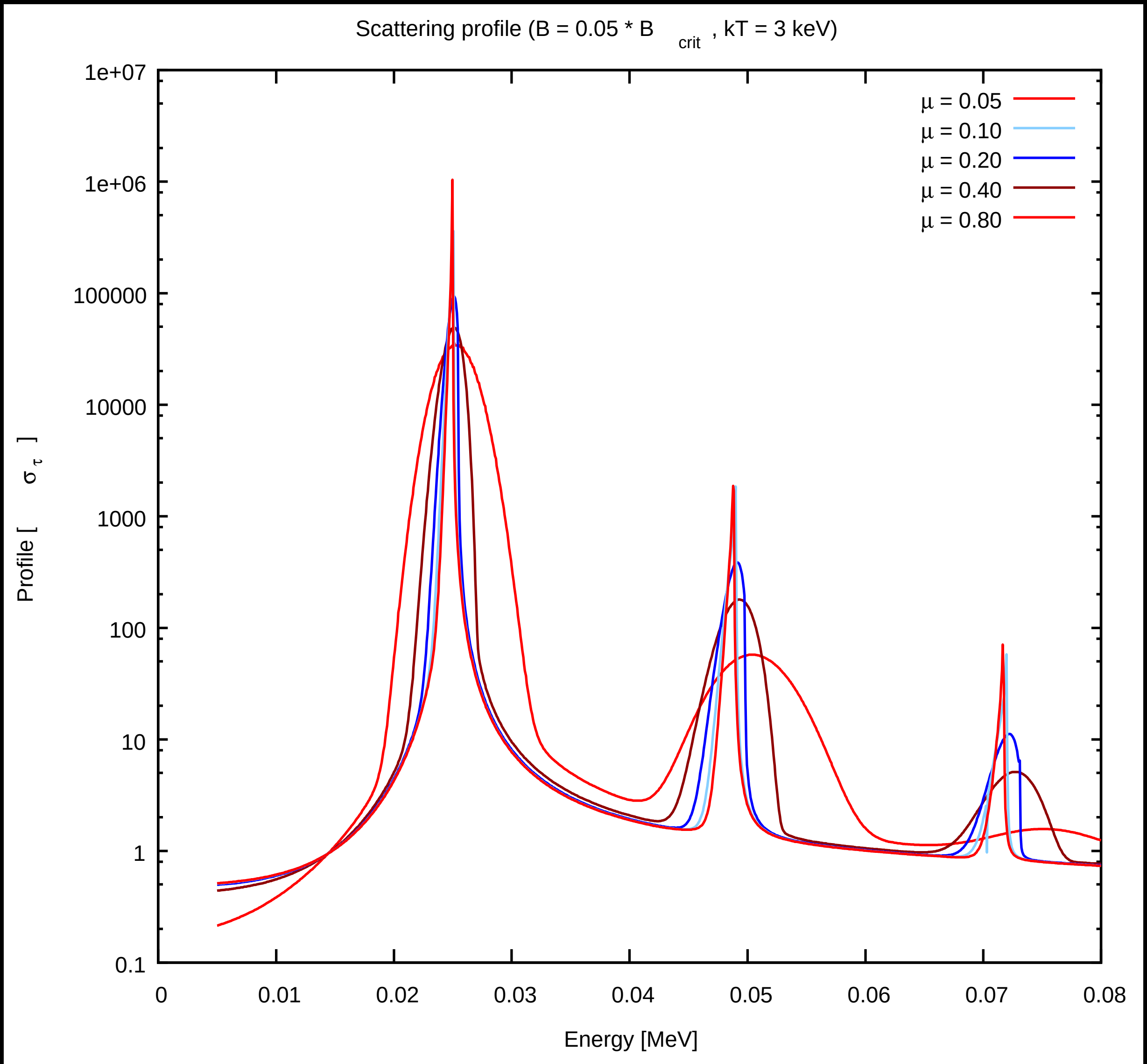
($12 - B_{12}$-rule)

$\Longrightarrow$ Cyclotron Resonance Scattering Features ("Cyclotron lines") at

$$E_n = nE_{\mathrm{cyc}} = (1+z)E_{n,\mathrm{obs}}$$

($1 + z \sim 1.25 \ldots 1.4$; grav. redshift!)

First discovery: Trümper et al. (1978)



Hot plasma

$\Longrightarrow$ thermal broadening:

- Lines narrow perpendicular to B-field
- Lines broad for motion along B-field

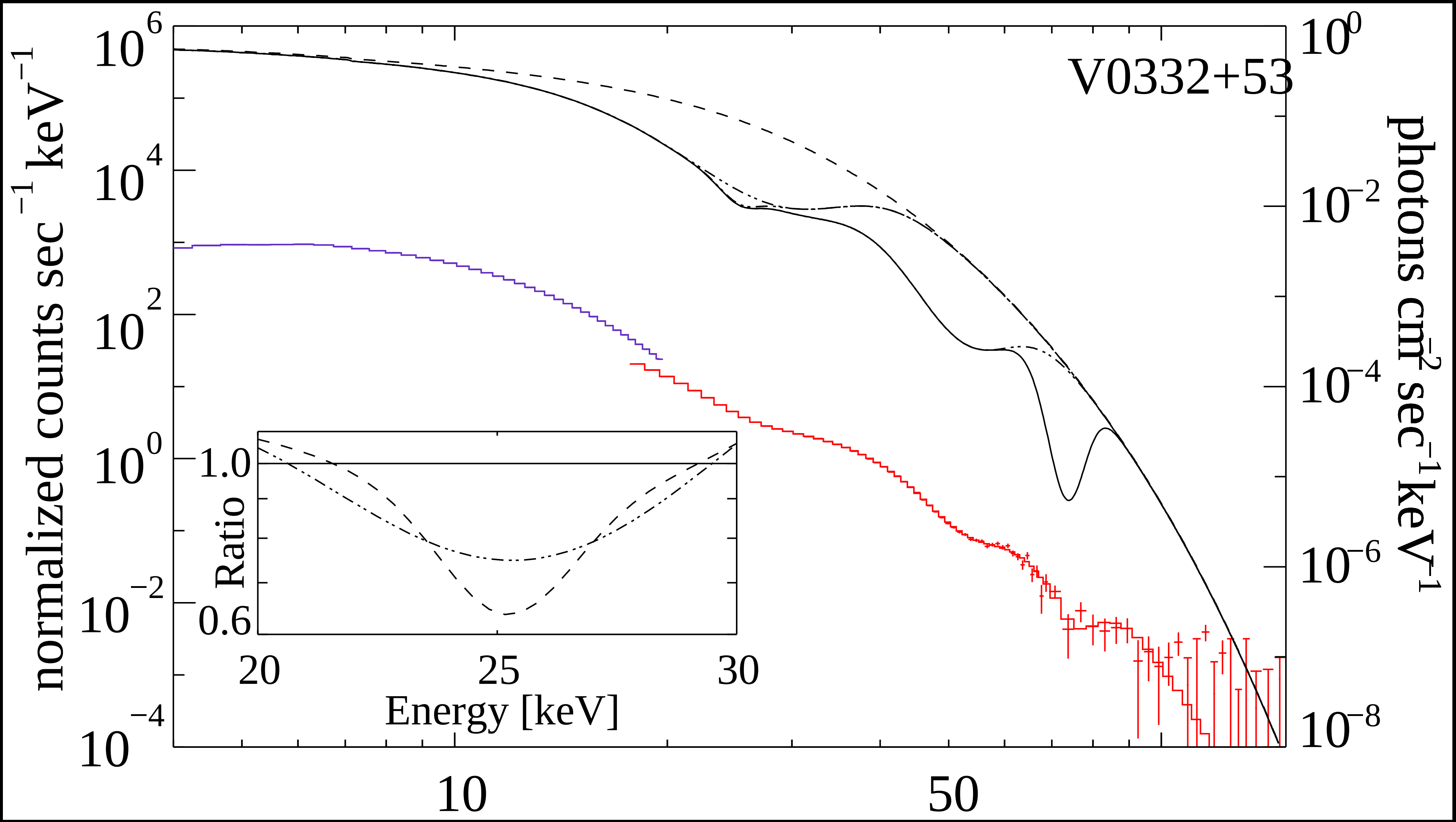
Expected line width

$$\frac{\Delta E_{\text{FWHM}}}{E_{\text{cyc}}} \sim \sqrt{kT_{\text{e}}} |\cos\theta|$$

($\sim$ 6 keV for $kT_{\text{e}} = 40$ keV)

Truemper et al. (1978), Meszaros (1992)

$B = 2.2 \times 10^{12}$ G,
θ= angle between B-field and photon direction; Schwarm (priv. comm.)

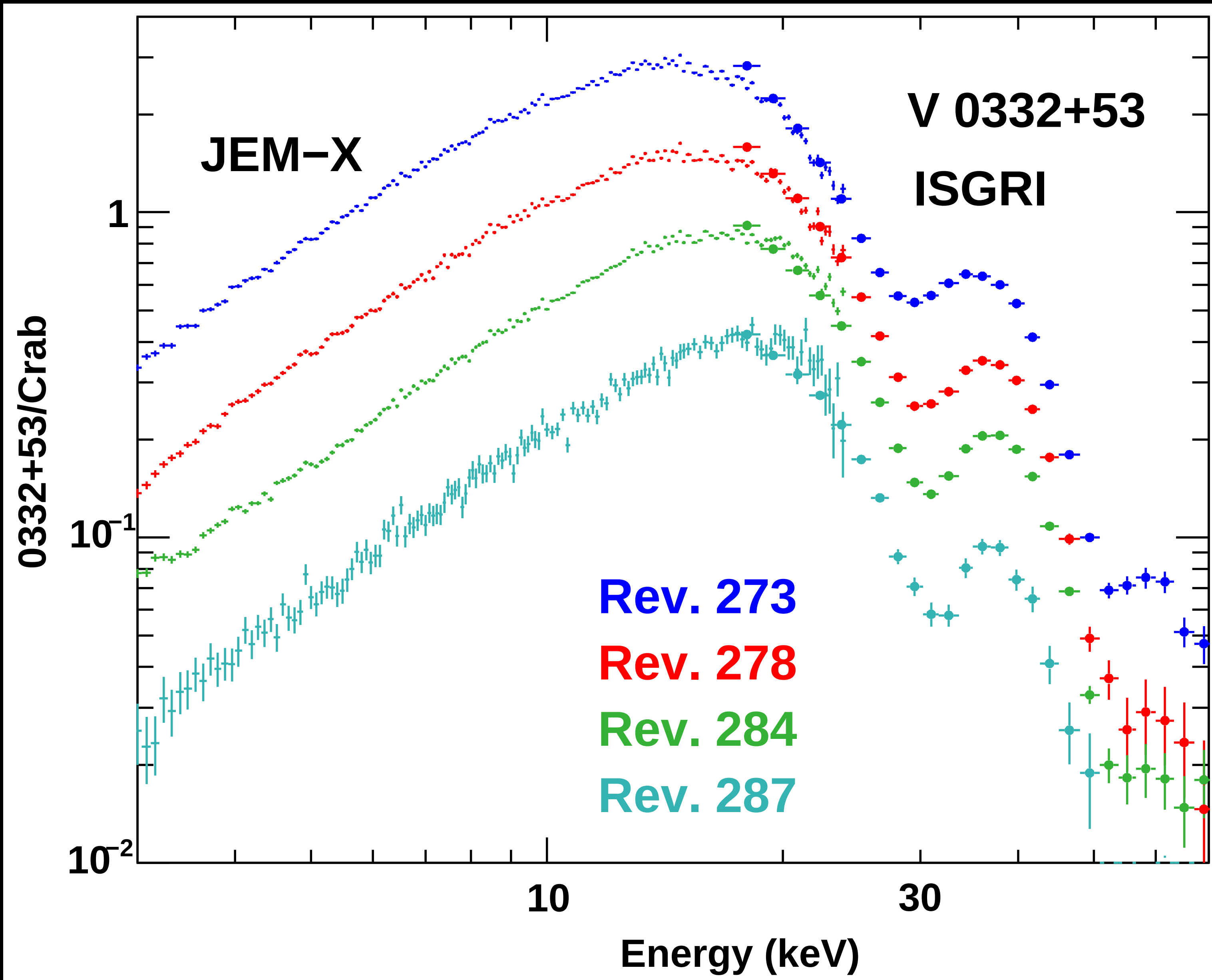


Pottschmidt et al. (2005)

V0332+53: Cyclotron lines at 27, 51, and 74 keV; complex fundamental.

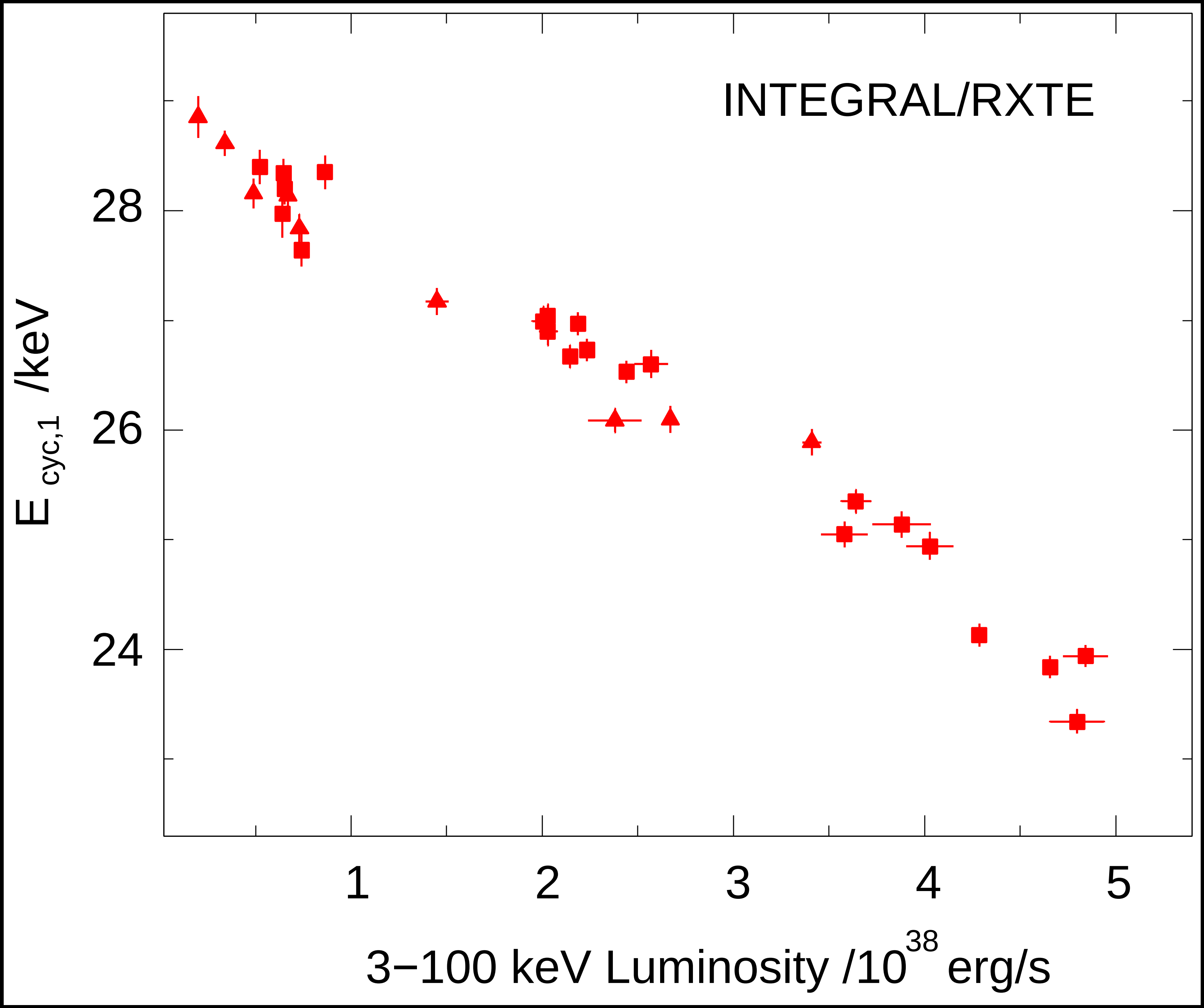
2nd source after 4U 0115+63 with more than 2 lines.

Line ratios $\neq$ 2, agrees with QED prediction; also require scattering angle of $\gtrsim$ 60°, in agreement with expectation from resonant cross-section.



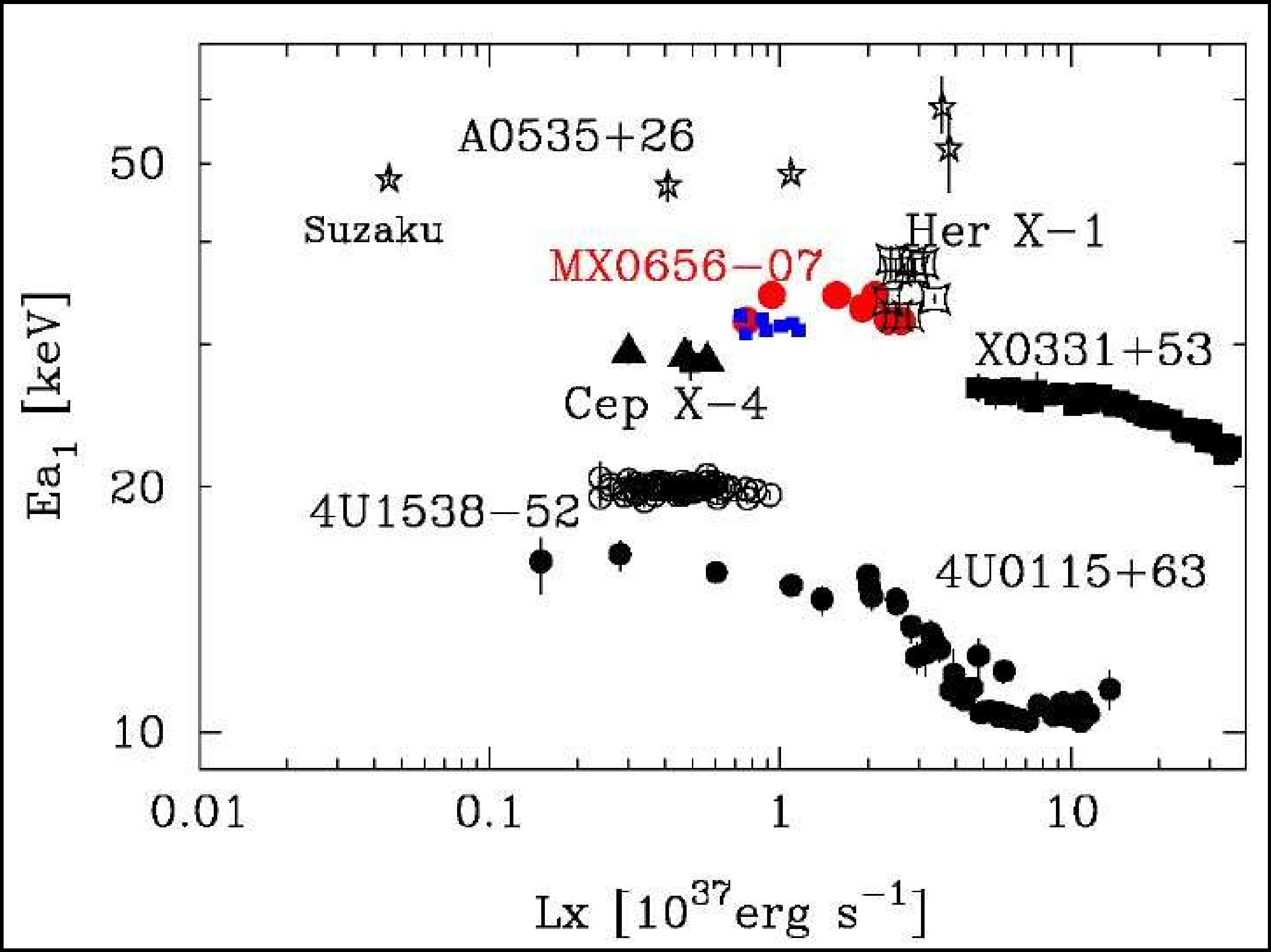
V0332+53: Energy of fundamental cyclotron line changes over outburst

Mowlavi et al. (2006)



(Tsygankov et al., 2006)

V0332+53: Cyclotron line energy depends on luminosity



Variation of cyclotron line is probably caused by interaction of ram pressure of accretion stream and radiation pressure in two different luminosity regimes.

(Mihara et al., 2007, Nakajima, 2008, Dauser, 2008)

The End